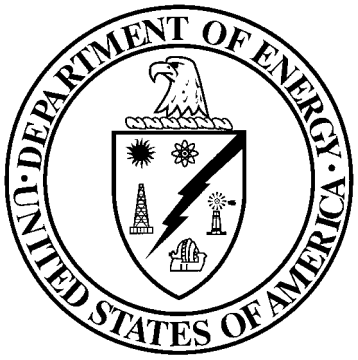
DOE-HDBK-1019/1-93
JANUARY 1993

DOE FUNDAMENTALS HANDBOOK

NUCLEAR PHYSICS AND REACTOR THEORY

Volume 1 of 2



U.S. Department of Energy **FSC-6910**
Washington, D.C. 20585

Distribution Statement A. Approved for public release; distribution is unlimited.

This document has been reproduced directly from the best available copy.

Available to DOE and DOE contractors from the Office of Scientific and Technical Information. P.O. Box 62, Oak Ridge, TN 37831.

Available to the public from the National Technical Information Services, U.S. Department of Commerce, 5285 Port Royal., Springfield, VA 22161.

Order No. DE93012221

ABSTRACT

The *Nuclear Physics and Reactor Theory* Handbook was developed to assist nuclear facility operating contractors in providing operators, maintenance personnel, and the technical staff with the necessary fundamentals training to ensure a basic understanding of nuclear physics and reactor theory. The handbook includes information on atomic and nuclear physics; neutron characteristics; reactor theory and nuclear parameters; and the theory of reactor operation. This information will provide personnel with a foundation for understanding the scientific principles that are associated with various DOE nuclear facility operations and maintenance.

Key Words: Training Material, Atomic Physics, The Chart of the Nuclides, Radioactivity, Radioactive Decay, Neutron Interaction, Fission, Reactor Theory, Neutron Characteristics, Neutron Life Cycle, Reactor Kinetics

FOREWORD

The *Department of Energy (DOE) Fundamentals Handbooks* consist of ten academic subjects, which include Mathematics; Classical Physics; Thermodynamics, Heat Transfer, and Fluid Flow; Instrumentation and Control; Electrical Science; Material Science; Mechanical Science; Chemistry; Engineering Symbology, Prints, and Drawings; and Nuclear Physics and Reactor Theory. The handbooks are provided as an aid to DOE nuclear facility contractors.

These handbooks were first published as Reactor Operator Fundamentals Manuals in 1985 for use by DOE category A reactors. The subject areas, subject matter content, and level of detail of the Reactor Operator Fundamentals Manuals were determined from several sources. DOE Category A reactor training managers determined which materials should be included, and served as a primary reference in the initial development phase. Training guidelines from the commercial nuclear power industry, results of job and task analyses, and independent input from contractors and operations-oriented personnel were all considered and included to some degree in developing the text material and learning objectives.

The *DOE Fundamentals Handbooks* represent the needs of various DOE nuclear facilities' fundamental training requirements. To increase their applicability to nonreactor nuclear facilities, the Reactor Operator Fundamentals Manual learning objectives were distributed to the Nuclear Facility Training Coordination Program Steering Committee for review and comment. To update their reactor-specific content, DOE Category A reactor training managers also reviewed and commented on the content. On the basis of feedback from these sources, information that applied to two or more DOE nuclear facilities was considered generic and was included. The final draft of each of the handbooks was then reviewed by these two groups. This approach has resulted in revised modular handbooks that contain sufficient detail such that each facility may adjust the content to fit their specific needs.

Each handbook contains an abstract, a foreword, an overview, learning objectives, and text material, and is divided into modules so that content and order may be modified by individual DOE contractors to suit their specific training needs. Each handbook is supported by a separate examination bank with an answer key.

The *DOE Fundamentals Handbooks* have been prepared for the Assistant Secretary for Nuclear Energy, Office of Nuclear Safety Policy and Standards, by the DOE Training Coordination Program. This program is managed by EG&G Idaho, Inc.

OVERVIEW

The *Department of Energy Fundamentals Handbook* entitled *Nuclear Physics and Reactor Theory* was prepared as an information resource for personnel who are responsible for the operation of the Department's nuclear facilities. Almost all processes that take place in a nuclear facility involves the transfer of some type of energy. A basic understanding of nuclear physics and reactor theory is necessary for DOE nuclear facility operators, maintenance personnel, and the technical staff to safely operate and maintain the facility and facility support systems. The information in this handbook is presented to provide a foundation for applying engineering concepts to the job. This knowledge will help personnel understand the impact that their actions may have on the safe and reliable operation of facility components and systems.

The *Nuclear Physics and Reactor Theory* handbook consists of four modules that are contained in two volumes. The following is a brief description of the information presented in each module of the handbook.

Volume 1 of 2

Module 1 - Atomic and Nuclear Physics

Introduces concepts of atomic physics including the atomic nature of matter, the chart of the nuclides, radioactivity and radioactive decay, neutron interactions and fission, and the interaction of radiation with matter.

Module 2 - Reactor Theory (Nuclear Parameters)

Provides information on reactor theory and neutron characteristics. Includes topics such as neutron sources, neutron flux, neutron cross sections, reaction rates, neutron moderation, and prompt and delayed neutrons.

OVERVIEW (Cont.)

Volume 2 of 2

Module 3 - Reactor Theory (Nuclear Parameters)

Explains the nuclear parameters associated with reactor theory. Topics include the neutron life cycle, reactivity and reactivity coefficients, neutron poisons, and control rods.

Module 4 - Reactor Theory (Reactor Operations)

Introduces the reactor operations aspect of reactor theory. Topics include subcritical multiplication, reactor kinetics, and reactor operation.

The information contained in this handbook is not all-encompassing. An attempt to present the entire subject of nuclear physics and reactor theory would be impractical. However, the *Nuclear Physics and Reactor Theory* handbook presents enough information to provide the reader with the fundamental knowledge necessary to understand the advanced theoretical concepts presented in other subject areas, and to understand basic system and equipment operation.

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY

Module 1

Atomic and Nuclear Physics

TABLE OF CONTENTS

LIST OF FIGURES iv

LIST OF TABLES v

REFERENCES vi

OBJECTIVES vii

ATOMIC NATURE OF MATTER 1

- Structure of Matter 1
- Subatomic Particles 2
- Bohr Model of the Atom 3
- Measuring Units on the Atomic Scale 4
- Nuclides 4
- Isotopes 6
- Atomic and Nuclear Radii 6
- Nuclear Forces 7
- Summary 9

CHART OF THE NUCLIDES 11

- Chart of the Nuclides 11
- Information for Stable Nuclides 13
- Information for Unstable Nuclides 13
- Neutron - Proton Ratios 14
- Natural Abundance of Isotopes 15
- Enriched and Depleted Uranium 15
- Summary 16

MASS DEFECT AND BINDING ENERGY 17

- Mass Defect 17
- Binding Energy 18
- Energy Levels of Atoms 19
- Energy Levels of the Nucleus 20
- Summary 21

TABLE OF CONTENTS (Cont.)

MODES OF RADIOACTIVE DECAY 22

Stability of Nuclei 22
Natural Radioactivity 22
Nuclear Decay 23
Alpha Decay (α) 24
Beta Decay (β) 24
Electron Capture (EC, K-capture) 25
Gamma Emission (γ) 26
Internal Conversion 26
Isomers and Isomeric Transition 26
Decay Chains 27
Predicting Type of Decay 27
Summary 29

RADIOACTIVITY 30

Radioactive Decay Rates 30
Units of Measurement for Radioactivity 31
Variation of Radioactivity Over Time 31
Radioactive Half-Life 32
Plotting Radioactive Decay 35
Radioactive Equilibrium 38
Transient Radioactive Equilibrium 40
Summary 41

NEUTRON INTERACTIONS 43

Scattering 43
Elastic Scattering 43
Inelastic Scattering 45
Absorption Reactions 46
Radiative Capture 46
Particle Ejection 46
Fission 46
Summary 47

TABLE OF CONTENTS (Cont.)

NUCLEAR FISSION 48

Fission 48
Liquid Drop Model of a Nucleus 49
Critical Energy 50
Fissile Material 50
Fissionable Material 51
Fertile Material 52
Binding Energy Per Nucleon (BE/A) 53
Summary 54

ENERGY RELEASE FROM FISSION 56

Calculation of Fission Energy 56
Estimation of Decay Energy 60
Distribution of Fission Energy 61
Summary 62

INTERACTION OF RADIATION WITH MATTER 63

Interaction of Radiation With Matter 63
Alpha Radiation 64
Beta Minus Radiation 64
Positron Radiation 65
Neutron Radiation 65
Gamma Radiation 66
Summary 67

LIST OF FIGURES

Figure 1 Bohr's Model of the Hydrogen Atom 3

Figure 2 Nomenclature for Identifying Nuclides 5

Figure 3 Nuclide Chart for Atomic Numbers 1 to 6 12

Figure 4 Stable Nuclides 13

Figure 5 Unstable Nuclides 13

Figure 6 Neutron - Proton Plot of the Stable Nuclides 14

Figure 7 Energy Level Diagram - Nickel-60 20

Figure 8 Orbital Electron Capture 25

Figure 9 Types of Radioactive Decay Relative to the Line of Stability 28

Figure 10 Radioactive Decay as a Function of Time in Units of Half-Life 33

Figure 11 Linear and Semi-Log Plots of Nitrogen-16 Decay 37

Figure 12 Combined Decay of Iron-56, Manganese-54, and Cobalt-60 38

Figure 13 Cumulative Production of Sodium-24 Over Time 39

Figure 14 Approach of Sodium-24 to Equilibrium 40

Figure 15 Transient Equilibrium in the Decay of Barium-140 41

Figure 16 Elastic Scattering 44

Figure 17 Inelastic Scattering 45

Figure 18 Liquid Drop Model of Fission 50

Figure 19 Conversion of Fertile Nuclides to Fissile Nuclides 52

Figure 20 Binding Energy per Nucleon vs. Mass Number 53

Figure 21 Uranium-235 Fission Yield vs. Mass Number 57

Figure 22 Change in Binding Energy for Typical Fission 58

LIST OF TABLES

Table 1 Properties of Subatomic Particles 4

Table 2 Calculated Values for Nuclear Radii 7

Table 3 Forces Acting in the Nucleus 9

Table 4 Critical Energies Compared to Binding Energy of Last Neutron 51

Table 5 Binding Energies Calculated from Binding Energy per Nucleon Curve 58

Table 6 Instantaneous Energy from Fission 61

Table 7 Delayed Energy from Fission 61

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F.J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Given sufficient information, **DESCRIBE** atoms, including components, structure, and nomenclature.

ENABLING OBJECTIVES

1.1 **STATE** the characteristics of the following atomic particles, including mass, charge, and location within the atom:

a. Proton
b. Neutron
c. Electron

1.2 **DESCRIBE** the Bohr model of an atom.

1.3 **DEFINE** the following terms:

a. Nuclide
b. Isotope
c. Atomic number
d. Mass number

1.4 Given the standard ${}^{A}_{Z}X$ notation for a particular nuclide, **DETERMINE** the following:

a. Number of protons
b. Number of neutrons
c. Number of electrons

1.5 **DESCRIBE** the three forces that act on particles within the nucleus and affect the stability of the nucleus.

1.6 **DEFINE** the following terms:

a. Enriched uranium
b. Depleted uranium

1.7 **DEFINE** the following terms:

a. Mass defect
b. Binding energy

1.8 Given the atomic mass for a nuclide and the atomic masses of a neutron, proton, and electron, **CALCULATE** the mass defect and binding energy of the nuclide.

TERMINAL OBJECTIVE

2.0 Given necessary references, **DESCRIBE** the various modes of radioactive decay.

ENABLING OBJECTIVES

2.1 **DESCRIBE** the following processes:

a. Alpha decay
b. Beta-minus decay
c. Beta-plus decay
d. Electron capture
e. Internal conversions
f. Isomeric transitions

2.2 Given a Chart of the Nuclides, **WRITE** the radioactive decay chain for a nuclide.

2.3 **EXPLAIN** why one or more gamma rays typically accompany particle emission.

2.4 Given the stability curve on the Chart of the Nuclides, **DETERMINE** the type of radioactive decay that the nuclides in each region of the chart will typically undergo.

2.5 **DEFINE** the following terms:

a. Radioactivity
b. Curie
c. Becquerel
d. Radioactive decay constant
e. Radioactive half-life

2.6 Given the number of atoms and either the half-life or decay constant of a nuclide, **CALCULATE** the activity.

2.7 Given the initial activity and the decay constant of a nuclide, **CALCULATE** the activity at any later time.

2.8 **CONVERT** between the half-life and decay constant for a nuclide.

2.9 Given the Chart of the Nuclides and the original activity, **PLOT** the radioactive decay curve for a nuclide on either linear or semi-log coordinates.

2.10 **DEFINE** the following terms:

a. Radioactive equilibrium
b. Transient radioactive equilibrium

TERMINAL OBJECTIVE

3.0 Without references, **DESCRIBE** the different nuclear interactions initiated by neutrons.

ENABLING OBJECTIVES

3.1 **DESCRIBE** the following scattering interactions between a neutron and a nucleus:

a. Elastic scattering
b. Inelastic scattering

3.2 **STATE** the conservation laws that apply to an elastic collision between a neutron and a nucleus.

3.3 **DESCRIBE** the following reactions where a neutron is absorbed in a nucleus:

a. Radiative capture
b. Particle ejection

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the fission process.

ENABLING OBJECTIVES

4.1 **EXPLAIN** the fission process using the liquid drop model of a nucleus.

4.2 **DEFINE** the following terms:

a. Excitation energy
b. Critical energy

4.3 **DEFINE** the following terms:

a. Fissile material
b. Fissionable material
c. Fertile material

4.4 **DESCRIBE** the processes of transmutation, conversion, and breeding.

4.5 **DESCRIBE** the curve of Binding Energy per Nucleon versus mass number and give a qualitative description of the reasons for its shape.

4.6 **EXPLAIN** why only the heaviest nuclei are easily fissioned.

4.7 **EXPLAIN** why uranium-235 fissions with thermal neutrons and uranium-238 fissions only with fast neutrons.

4.8 **CHARACTERIZE** the fission products in terms of mass groupings and radioactivity.

4.9 Given the nuclides involved and their masses, **CALCULATE** the energy released from fission.

4.10 Given the curve of Binding Energy per Nucleon versus mass number, **CALCULATE** the energy released from fission.

TERMINAL OBJECTIVE

5.0 Without references, **DESCRIBE** how the various types of radiation interact with matter.

ENABLING OBJECTIVES

5.1 **DESCRIBE** interactions of the following with matter:

a. Alpha particle
b. Beta particle
c. Positron
d. Neutron

5.2 **DESCRIBE** the following ways that gamma radiation interacts with matter:

a. Compton scattering
b. Photoelectric effect
c. Pair production

Intentionally Left Blank

ATOMIC NATURE OF MATTER

All matter is composed of atoms. The atom is the smallest amount of matter that retains the properties of an element. Atoms themselves are composed of smaller particles, but these smaller particles no longer have the same properties as the overall element.

EO 1.1 STATE the characteristics of the following atomic particles, including mass, charge, and location within the atom:

a. Proton
b. Neutron
c. Electron

EO 1.2 DESCRIBE the Bohr model of an atom.

EO 1.3 DEFINE the following terms:

a. Nuclide
b. Isotope
c. Atomic number
d. Mass number

EO 1.4 Given the standard $^{A}_{Z}X$ notation for a particular nuclide, DETERMINE the following:

a. Number of protons
b. Number of neutrons
c. Number of electrons

EO 1.5 DESCRIBE the three forces that act on particles within the nucleus and affect the stability of the nucleus.

Structure of Matter

Early Greek philosophers speculated that the earth was made up of different combinations of basic substances, or elements. They considered these basic elements to be earth, air, water, and fire. Modern science shows that the early Greeks held the correct concept that matter consists of a combination of basic elements, but they incorrectly identified the elements.

In 1661 the English chemist Robert Boyle published the modern criterion for an element. He defined an element to be a basic substance that cannot be broken down into any simpler substance after it is isolated from a compound, but can be combined with other elements to form compounds. To date, 105 different elements have been confirmed to exist, and researchers claim to have discovered three additional elements. Of the 105 confirmed elements, 90 exist in nature and 15 are man-made.

Another basic concept of matter that the Greeks debated was whether matter was continuous or discrete. That is, whether matter could be continuously divided and subdivided into ever smaller particles or whether eventually an indivisible particle would be encountered. Democritus in about 450 B.C. argued that substances were ultimately composed of small, indivisible particles that he labeled atoms. He further suggested that different substances were composed of different atoms or combinations of atoms, and that one substance could be converted into another by rearranging the atoms. It was impossible to conclusively prove or disprove this proposal for more than 2000 years.

The modern proof for the atomic nature of matter was first proposed by the English chemist John Dalton in 1803. Dalton stated that each chemical element possesses a particular kind of atom, and any quantity of the element is made up of identical atoms of this kind. What distinguishes one element from another element is the kind of atom of which it consists, and the basic physical difference between kinds of atoms is their weight.

Subatomic Particles

For almost 100 years after Dalton established the atomic nature of atoms, it was considered impossible to divide the atom into even smaller parts. All of the results of chemical experiments during this time indicated that the atom was indivisible. Eventually, experimentation into electricity and radioactivity indicated that particles of matter smaller than the atom did indeed exist. In 1906, J. J. Thompson won the Nobel Prize in physics for establishing the existence of electrons. *Electrons* are negatively-charged particles that have 1/1835 the mass of the hydrogen atom. Soon after the discovery of electrons, protons were discovered. *Protons* are relatively large particles that have almost the same mass as a hydrogen atom and a positive charge equal in magnitude (but opposite in sign) to that of the electron. The third subatomic particle to be discovered, the neutron, was not found until 1932. The *neutron* has almost the same mass as the proton, but it is electrically neutral.

Bohr Model of the Atom

The British physicist Ernest Rutherford postulated that the positive charge in an atom is concentrated in a small region called a nucleus at the center of the atom with electrons existing in orbits around it. Niels Bohr, coupling Rutherford's postulation with the quantum theory introduced by Max Planck, proposed that the atom consists of a dense nucleus of protons surrounded by electrons traveling in discrete orbits at fixed distances from the nucleus. An electron in one of these orbits or shells has a specific or discrete quantity of energy (quantum). When an electron moves from one allowed orbit to another allowed orbit, the energy difference between the two states is emitted or absorbed in the form of a single quantum of radiant energy called a photon. Figure 1 is Bohr's model of the hydrogen atom showing an electron as having just dropped from the third shell to the first shell with the emission of a photon that has an energy = hν. (h = Planck's constant = 6.63 x 10^{-34} J-s and ν = frequency of the photon.) Bohr's theory was the first to successfully account for the discrete energy levels of this radiation as measured in the laboratory. Although Bohr's atomic model is designed specifically to explain the hydrogen atom, his theories apply generally to the structure of all atoms. Additional information on electron shell theory can be found in the Chemistry Fundamentals Handbook.

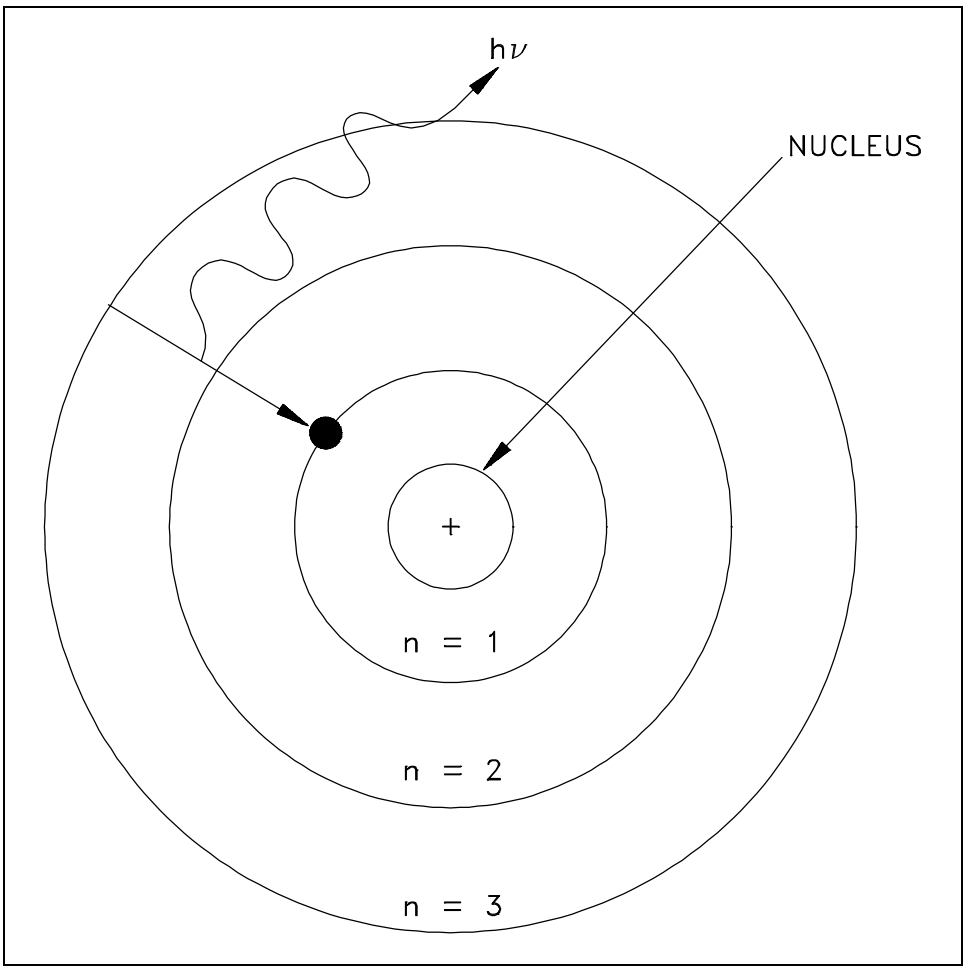


Figure 1 Bohr's Model of the Hydrogen Atom

Properties of the three subatomic particles are listed in Table 1.

TABLE 1
Properties of Subatomic Particles

Particle	Location	Charge	Mass
Neutron	Nucleus	none	1.008665 amu
Proton	Nucleus	+1	1.007277 amu
Electron	Shells around nucleus	-1	0.0005486 amu

Measuring Units on the Atomic Scale

The size and mass of atoms are so small that the use of normal measuring units, while possible, is often inconvenient. Units of measure have been defined for mass and energy on the atomic scale to make measurements more convenient to express. The unit of measure for mass is the atomic mass unit (amu). One atomic mass unit is equal to 1.66×10^{-24} grams. The reason for this particular value for the atomic mass unit will be discussed in a later chapter. Note from Table 1 that the mass of a neutron and a proton are both about 1 amu. The unit for energy is the electron volt (eV). The electron volt is the amount of energy acquired by a single electron when it falls through a potential difference of one volt. One electron volt is equivalent to 1.602×10^{-19} joules or 1.18×10^{-19} foot-pounds.

Nuclides

The total number of protons in the nucleus of an atom is called the *atomic number* of the atom and is given the symbol Z. The number of electrons in an electrically-neutral atom is the same as the number of protons in the nucleus. The number of neutrons in a nucleus is known as the neutron number and is given the symbol N. The *mass number* of the nucleus is the total number of nucleons, that is, protons and neutrons in the nucleus. The mass number is given the symbol A and can be found by the equation $Z + N = A$.

Each of the chemical elements has a unique atomic number because the atoms of different elements contain a different number of protons. The atomic number of an atom identifies the particular element.

Each type of atom that contains a unique combination of protons and neutrons is called a *nuclide*. Not all combinations of numbers of protons and neutrons are possible, but about 2500 specific nuclides with unique combinations of neutrons and protons have been identified. Each nuclide is denoted by the chemical symbol of the element with the atomic number written as a subscript and the mass number written as a superscript, as shown in Figure 2. Because each element has a unique name, chemical symbol, and atomic number, only one of the three is necessary to identify the element. For this reason nuclides can also be identified by either the chemical name or the chemical symbol followed by the mass number (for example, U-235 or uranium-235). Another common format is to use the abbreviation of the chemical element with the mass number superscripted (for example, ^{235}U). In this handbook the format used in the text will usually be the element's name followed by the mass number. In equations and tables, the format in Figure 2 will usually be used.

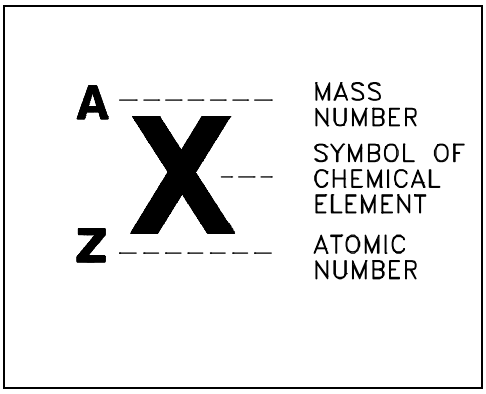


Figure 2 Nomenclature for Identifying Nuclides

Example:

State the name of the element and the number of protons, electrons, and neutrons in the nuclides listed below.

$^{1}_{1}$H

$^{10}_{5}$B

$^{14}_{7}$N

$^{114}_{48}$Cd

$^{239}_{94}$Pu

Solution:

The name of the element can be found from the Periodic Table (refer to Chemistry Fundamentals Handbook) or the Chart of the Nuclides (to be discussed later). The number of protons and electrons are equal to Z. The number of neutrons is equal to Z - A.

Nuclide	Element	Protons	Electrons	Neutrons
$^{1}_{1}H$	hydrogen	1	1	0
$^{10}_{5}B$	boron	5	5	5
$^{14}_{7}N$	nitrogen	7	7	7
$^{114}_{48}Cd$	cadmium	48	48	66
$^{239}_{94}Pu$	plutonium	94	94	145

Isotopes

Isotopes are nuclides that have the same atomic number and are therefore the same element, but differ in the number of neutrons. Most elements have a few stable isotopes and several unstable, radioactive isotopes. For example, oxygen has three stable isotopes that can be found in nature (oxygen-16, oxygen-17, and oxygen-18) and eight radioactive isotopes. Another example is hydrogen, which has two stable isotopes (hydrogen-1 and hydrogen-2) and a single radioactive isotope (hydrogen-3).

The isotopes of hydrogen are unique in that they are each commonly referred to by a unique name instead of the common chemical element name. Hydrogen-1 is almost always referred to as hydrogen, but the term protium is infrequently used also. Hydrogen-2 is commonly called deuterium and symbolized $^{2}_{1}D$. Hydrogen-3 is commonly called tritium and symbolized $^{3}_{1}T$. This text will normally use the symbology $^{2}_{1}H$ and $^{3}_{1}H$ for deuterium and tritium, respectively.

Atomic and Nuclear Radii

The size of an atom is difficult to define exactly due to the fact that the electron cloud, formed by the electrons moving in their various orbitals, does not have a distinct outer edge. A reasonable measure of atomic size is given by the average distance of the outermost electron from the nucleus. Except for a few of the lightest atoms, the average atomic radii are approximately the same for all atoms, about 2×10^{-8} cm.

Like the atom the nucleus does not have a sharp outer boundary. Experiments have shown that the nucleus is shaped like a sphere with a radius that depends on the atomic mass number of the atom. The relationship between the atomic mass number and the radius of the nucleus is shown in the following equation.

$$r = (1.25 \times 10^{-13} \text{ cm}) A^{1/3}$$

where:

r = radius of the nucleus (cm)
A = atomic mass number (dimensionless)

The values of the nuclear radii for some light, intermediate, and heavy nuclides are shown in Table 2.

TABLE 2
Calculated Values for Nuclear Radii

Nuclide	Radius of Nucleus
$^{1}_{1}H$	1.25×10^{-13} cm
$^{10}_{5}B$	2.69×10^{-13} cm
$^{56}_{26}Fe$	4.78×10^{-13} cm
$^{178}_{72}Hf$	7.01×10^{-13} cm
$^{238}_{92}U$	7.74×10^{-13} cm
$^{252}_{98}Cf$	7.89×10^{-13} cm

From the table, it is clear that the radius of a typical atom (e.g. 2×10^{-8} cm) is more than 25,000 times larger than the radius of the largest nucleus.

Nuclear Forces

In the Bohr model of the atom, the nucleus consists of positively-charged protons and electrically-neutral neutrons. Since both protons and neutrons exist in the nucleus, they are both referred to as nucleons. One problem that the Bohr model of the atom presented was accounting for an attractive force to overcome the repulsive force between protons.

Two forces present in the nucleus are (1) electrostatic forces between charged particles and (2) gravitational forces between any two objects that have mass. It is possible to calculate the magnitude of the gravitational force and electrostatic force based upon principles from classical physics.

Newton stated that the *gravitational force* between two bodies is directly proportional to the masses of the two bodies and inversely proportional to the square of the distance between the bodies. This relationship is shown in the equation below.

$$F_g = \frac{G\ m_1\ m_2}{r^2}$$

where:

F_g = gravitational force (newtons)
m_1 = mass of first body (kilograms)
m_2 = mass of second body (kilograms)
G = gravitational constant (6.67×10^{-11} N-m^2/kg^2)
r = distance between particles (meters)

The equation illustrates that the larger the masses of the objects or the smaller the distance between the objects, the greater the gravitational force. So even though the masses of nucleons are very small, the fact that the distance between nucleons is extremely short may make the gravitational force significant. It is necessary to calculate the value for the gravitational force and compare it to the value for other forces to determine the significance of the gravitational force in the nucleus. The gravitational force between two protons that are separated by a distance of 10^{-20} meters is about 10^{-24} newtons.

Coulomb's Law can be used to calculate the force between two protons. The *electrostatic force* is directly proportional to the electrical charges of the two particles and inversely proportional to the square of the distance between the particles. Coulomb's Law is stated as the following equation.

$$F_e = \frac{K\ Q_1\ Q_2}{r^2}$$

where:

F_e = electrostatic force (newtons)
K = electrostatic constant (9.0×10^9 N-m^2/C^2)
Q_1 = charge of first particle (coulombs)
Q_2 = charge of second particle (coulombs)
r = distance between particles (meters)

Using this equation, the electrostatic force between two protons that are separated by a distance of 10^{-20} meters is about 10^{12} newtons. Comparing this result with the calculation of the gravitational force (10^{-24} newtons) shows that the gravitational force is so small that it can be neglected.

If only the electrostatic and gravitational forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist, there must be another attractive force acting within the nucleus. This force is called the nuclear force.

The *nuclear force* is a strong attractive force that is independent of charge. It acts equally only between pairs of neutrons, pairs of protons, or a neutron and a proton. The nuclear force has a very short range; it acts only over distances approximately equal to the diameter of the nucleus (10^{-13} cm). The attractive nuclear force between all nucleons drops off with distance much faster than the repulsive electrostatic force between protons.

TABLE 3
Forces Acting in the Nucleus

Force	Interaction	Range
Gravitational	Very weak attractive force between all nucleons	Relatively long
Electrostatic	Strong repulsive force between like charged particles (protons)	Relatively long
Nuclear Force	Strong attractive force between all nucleons	Extremely short

In stable atoms, the attractive and repulsive forces in the nucleus balance. If the forces do not balance, the atom cannot be stable, and the nucleus will emit radiation in an attempt to achieve a more stable configuration.

Summary

The important information in this chapter is summarized on the following page.

Atomic Nature of Matter Summary

- Atoms consist of three basic subatomic particles. These particles are the proton, the neutron, and the electron.
- Protons are particles that have a positive charge, have about the same mass as a hydrogen atom, and exist in the nucleus of an atom.
- Neutrons are particles that have no electrical charge, have about the same mass as a hydrogen atom, and exist in the nucleus of an atom.
- Electrons are particles that have a negative charge, have a mass about eighteen hundred times smaller than the mass of a hydrogen atom, and exist in orbital shells around the nucleus of an atom.
- The Bohr model of the atom consists of a dense nucleus of protons and neutrons (nucleons) surrounded by electrons traveling in discrete orbits at fixed distances from the nucleus.
- Nuclides are atoms that contain a particular number of protons and neutrons.
- Isotopes are nuclides that have the same atomic number and are therefore the same element, but differ in the number of neutrons.
- The atomic number of an atom is the number of protons in the nucleus.
- The mass number of an atom is the total number of nucleons (protons and neutrons) in the nucleus.
- The notation ${}^{A}_{Z}X$ is used to identify a specific nuclide. "Z" represents the atomic number, which is equal to the number of protons. "A" represents the mass number, which is equal to the number of nucleons. "X" represents the chemical symbol of the element.

  Number of protons = Z
  Number of electrons = Z
  Number of neutrons = A - Z

- The stability of a nucleus is determined by the different forces interacting within it. The electrostatic force is a relatively long-range, strong, repulsive force that acts between the positively charged protons. The nuclear force is a relatively short-range attractive force between all nucleons. The gravitational force the long range, relatively weak attraction between masses, is negligible compared to the other forces.

CHART OF THE NUCLIDES

The Chart of the Nuclides, like the Periodic Table, is a convenient format for presenting a large amount of scientific information in an organized manner.

EO 1.6 DEFINE the following terms:

a. Enriched uranium
b. Depleted uranium

Chart of the Nuclides

A tabulated chart called the *Chart of the Nuclides* lists the stable and unstable nuclides in addition to pertinent information about each one. Figure 3 shows a small portion of a typical chart. This chart plots a box for each individual nuclide, with the number of protons (Z) on the vertical axis and the number of neutrons (N = A - Z) on the horizontal axis.

The completely gray squares indicate stable isotopes. Those in white squares are *artificially radioactive*, meaning that they are produced by artificial techniques and do not occur naturally. By consulting a complete chart, other types of isotopes can be found, such as naturally occurring radioactive types (but none are found in the region of the chart that is illustrated in Figure 3).

Located in the box on the far left of each horizontal row is general information about the element. The box contains the chemical symbol of the element in addition to the average atomic weight of the naturally occurring substance and the average thermal neutron absorption cross section, which will be discussed in a later module. The known isotopes (elements with the same atomic number Z but different mass number A) of each element are listed to the right.

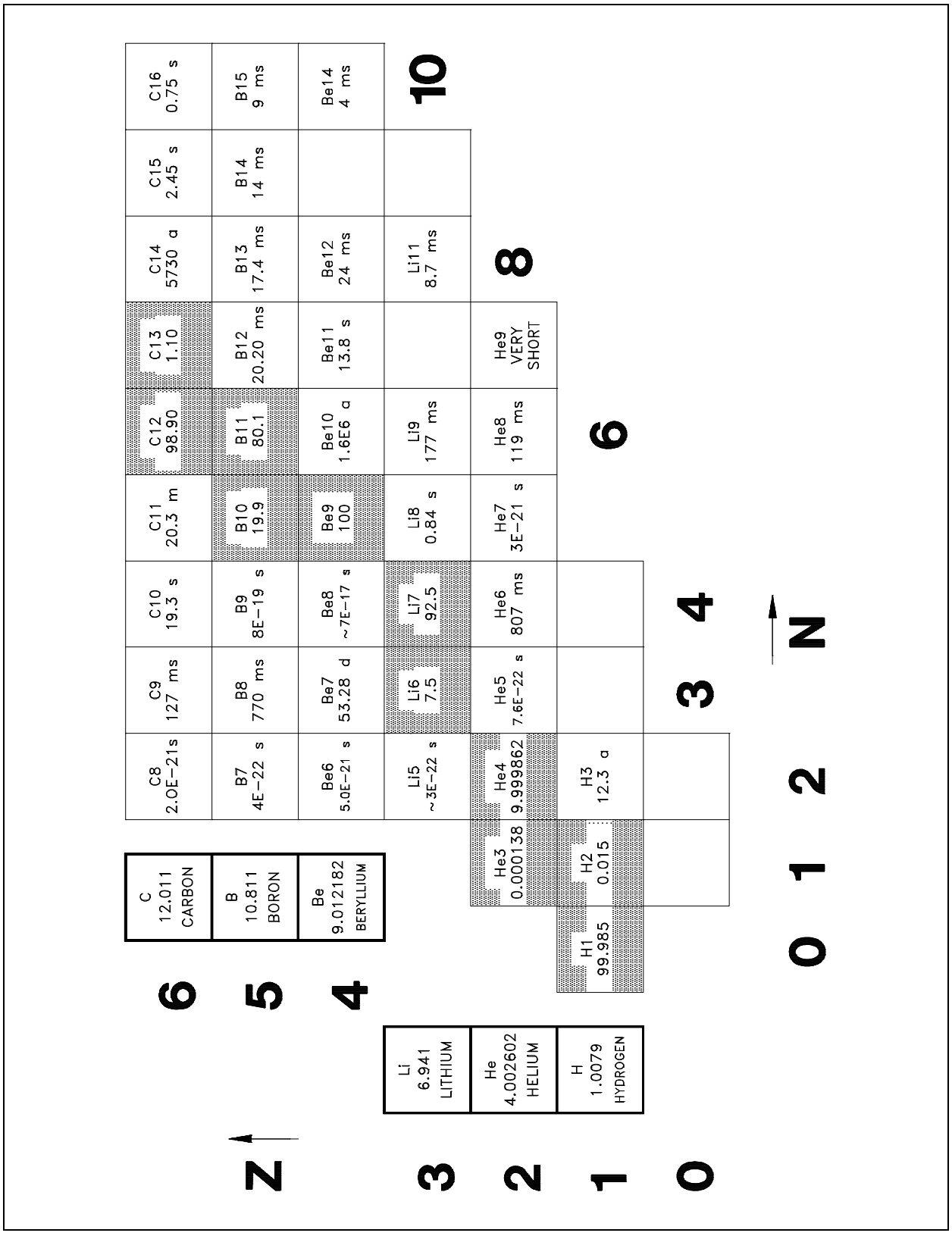


Figure 3 Nuclide Chart for Atomic Numbers 1 to 6

Information for Stable Nuclides

For the stable isotopes, in addition to the symbol and the atomic mass number, the number percentage of each isotope in the naturally occurring element is listed, as well as the thermal neutron activation cross section and the mass in atomic mass units (amu). A typical block for a stable nuclide from the Chart of the Nuclides is shown in Figure 4.

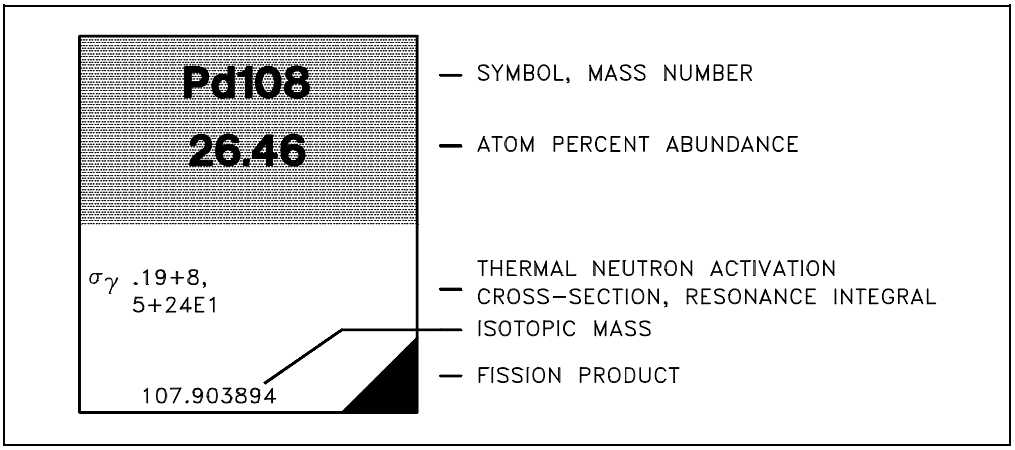


Figure 4 Stable Nuclides

Information for Unstable Nuclides

For unstable isotopes the additional information includes the half life, the mode of decay (for example, β^-, α), the total disintegration energy in MeV (million electron volts), and the mass in amu when available. A typical block for an unstable nuclide from the Chart of the Nuclides is shown in Figure 5.

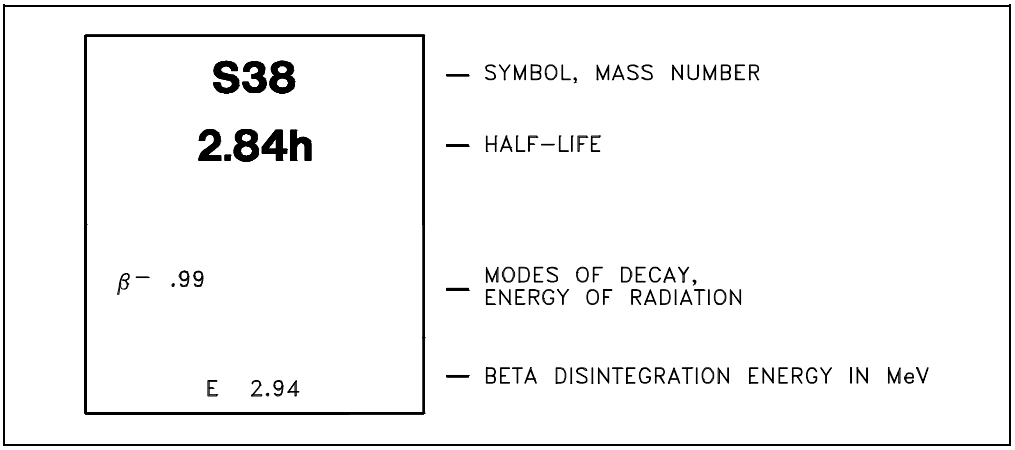


Figure 5 Unstable Nuclides

Neutron - Proton Ratios

Figure 6 shows the distribution of the stable nuclides plotted on the same axes as the Chart of the Nuclides. As the mass numbers become higher, the ratio of neutrons to protons in the nucleus becomes larger. For helium-4 (2 protons and 2 neutrons) and oxygen-16 (8 protons and 8 neutrons) this ratio is unity. For indium-115 (49 protons and 66 neutrons) the ratio of neutrons to protons has increased to 1.35, and for uranium-238 (92 protons and 146 neutrons) the neutron-to-proton ratio is 1.59.

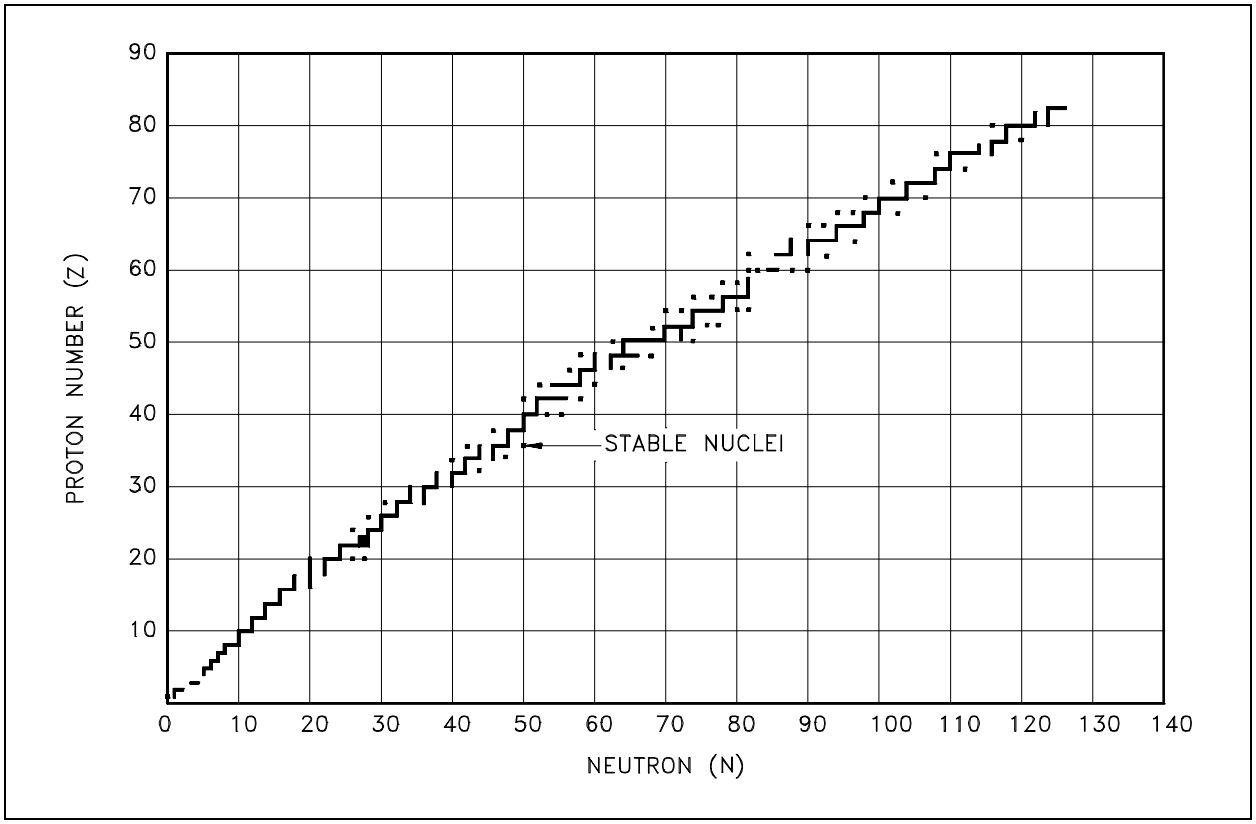


Figure 6 Neutron - Proton Plot of the Stable Nuclides

If a heavy nucleus were to split into two fragments, each fragment would form a nucleus that would have approximately the same neutron-to-proton ratio as the heavy nucleus. This high neutron-to-proton ratio places the fragments below and to the right of the stability curve displayed by Figure 6. The instability caused by this excess of neutrons is generally rectified by successive beta emissions, each of which converts a neutron to a proton and moves the nucleus toward a more stable neutron-to-proton ratio.

Natural Abundance of Isotopes

The relative abundance of an isotope in nature compared to other isotopes of the same element is relatively constant. The Chart of the Nuclides presents the relative abundance of the naturally occurring isotopes of an element in units of atom percent. Atom percent is the percentage of the atoms of an element that are of a particular isotope. Atom percent is abbreviated as a/o. For example, if a cup of water contains 8.23×10^{24} atoms of oxygen, and the isotopic abundance of oxygen-18 is 0.20%, then there are 1.65×10^{22} atoms of oxygen-18 in the cup.

The atomic weight for an element is defined as the average atomic weight of the isotopes of the element. The atomic weight for an element can be calculated by summing the products of the isotopic abundance of the isotope with the atomic mass of the isotope.

Example:

> Calculate the atomic weight for the element lithium. Lithium-6 has an atom percent abundance of 7.5% and an atomic mass of 6.015122 amu. Lithium-7 has an atomic abundance of 92.5% and an atomic mass of 7.016003 amu.

Solution:

$$\text{Atomic Mass Lithium} = (0.075)\ (6.015122\ \text{amu}) + (0.925)\ (7.016003\ \text{amu})$$
$$= 6.9409\ \text{amu}$$

The other common measurement of isotopic abundance is weight percent (w/o). Weight percent is the percent weight of an element that is a particular isotope. For example, if a sample of material contained 100 kg of uranium that was 28 w/o uranium-235, then 28 kg of uranium-235 was present in the sample.

Enriched and Depleted Uranium

Natural uranium mined from the earth contains the isotopes uranium-238, uranium-235 and uranium-234. The majority (99.2745%) of all the atoms in natural uranium are uranium-238. Most of the remaining atoms (0.72%) are uranium-235, and a slight trace (0.0055%) are uranium-234. Although all isotopes of uranium have similar chemical properties, each of the isotopes has significantly different nuclear properties. For reasons that will be discussed in later modules, the isotope uranium-235 is usually the desired material for use in reactors.

A vast amount of equipment and energy are expended in processes that separate the isotopes of uranium (and other elements). The details of these processes are beyond the scope of this module. These processes are called enrichment processes because they selectively increase the proportion of a particular isotope. The enrichment process typically starts with feed material that has the proportion of isotopes that occur naturally. The process results in two types of

In the case of uranium, the natural uranium ore is 0.72 a/o uranium-235. The desired outcome of the enrichment process is to produce enriched uranium. *Enriched uranium* is defined as uranium in which the isotope uranium-235 has a concentration greater than its natural value. The enrichment process will also result in the byproduct of depleted uranium. *Depleted uranium* is defined as uranium in which the isotope uranium-235 has a concentration less than its natural value. Although depleted uranium is referred to as a by-product of the enrichment process, it does have uses in the nuclear field and in commercial and defense industries.

Summary

The important information in this chapter is summarized below.

Chart of the Nuclides Summary

- Enriched uranium is uranium in which the isotope uranium-235 has a concentration greater than its natural value of 0.7%.
- Depleted uranium is uranium in which the isotope uranium-235 has a concentration less than its natural value of 0.7%.

MASS DEFECT AND BINDING ENERGY

The separate laws of Conservation of Mass and Conservation of Energy are not applied strictly on the nuclear level. It is possible to convert between mass and energy. Instead of two separate conservation laws, a single conservation law states that the sum of mass and energy is conserved. Mass does not magically appear and disappear at random. A decrease in mass will be accompanied by a corresponding increase in energy and vice versa.

EO 1.7 DEFINE the following terms:

a. Mass defect
b. Binding energy

EO 1.8 Given the atomic mass for a nuclide and the atomic masses of a neutron, proton, and electron, CALCULATE the mass defect and binding energy of the nuclide.

Mass Defect

Careful measurements have shown that the mass of a particular atom is always slightly less than the sum of the masses of the individual neutrons, protons, and electrons of which the atom consists. The difference between the mass of the atom and the sum of the masses of its parts is called the *mass defect* (Δm). The mass defect can be calculated using Equation (1-1). In calculating the mass defect it is important to use the full accuracy of mass measurements because the difference in mass is small compared to the mass of the atom. Rounding off the masses of atoms and particles to three or four significant digits prior to the calculation will result in a calculated mass defect of zero.

$$\Delta m = [\ Z(m_p + m_e) + (A-Z)m_n\ ] - m_{atom} \qquad (1\text{-}1)$$

where:

Δm = mass defect (amu)
m_p = mass of a proton (1.007277 amu)
m_n = mass of a neutron (1.008665 amu)
m_e = mass of an electron (0.000548597 amu)
m_{atom} = mass of nuclide ${}^{A}_{Z}X$ (amu)
Z = atomic number (number of protons)
A = mass number (number of nucleons)

Example:

Calculate the mass defect for lithium-7. The mass of lithium-7 is 7.016003 amu.

Solution:

$$\Delta m = \left[Z\left(m_p + m_e\right) + \left(A - Z\right) m_n\right] - m_{atom}$$

$$\Delta m = \left[3\left(1.007826\ amu\right) + \left(7 - 3\right) 1.008665\ amu\right] - 7.016003\ amu$$

$$\Delta m = 0.0421335\ amu$$

Binding Energy

The loss in mass, or mass defect, is due to the conversion of mass to binding energy when the nucleus is formed. *Binding energy* is defined as the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles (nucleons). It can also be understood as the amount of energy that would be released if the nucleus was formed from the separate particles. Binding energy is the energy equivalent of the mass defect. Since the mass defect was converted to binding energy (BE) when the nucleus was formed, it is possible to calculate the binding energy using a conversion factor derived by the mass-energy relationship from Einstein's Theory of Relativity.

Einstein's famous equation relating mass and energy is $E = mc^2$ where c is the velocity of light ($c = 2.998 \times 10^8$ m/sec). The energy equivalent of 1 amu can be determined by inserting this quantity of mass into Einstein's equation and applying conversion factors.

$$E = m\,c^2$$

$$= 1\ amu \left(\frac{1.6606 \times 10^{-27}\ kg}{1\ amu}\right)\left(2.998 \times 10^8\ \frac{m}{sec}\right)^2\left(\frac{1\ N}{1\ \frac{kg\text{-}m}{sec^2}}\right)\left(\frac{1\ J}{1\ N\text{-}m}\right)$$

$$= 1.4924 \times 10^{-10}\ J\left(\frac{1\ MeV}{1.6022 \times 10^{-13}\ J}\right)$$

$$= 931.5\ MeV$$

Conversion Factors:

1 amu	=	1.6606×10^{-27} kg
1 newton	=	1 kg-m/sec^2
1 joule	=	1 newton-meter
1 MeV	=	1.6022×10^{-13} joules

Since 1 amu is equivalent to 931.5 MeV of energy, the binding energy can be calculated using Equation (1-2).

$$B.E. = \Delta m \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right) \tag{1-2}$$

Example:

Calculate the mass defect and binding energy for uranium-235. One uranium-235 atom has a mass of 235.043924 amu.

Solution:

Step 1: Calculate the mass defect using Equation (1-1).

$$\Delta m = \left[Z \left(m_p + m_e \right) + \left(A - Z \right) m_n \right] - m_{atom}$$

$$\Delta m = \left[92 \left(1.007826 \text{ amu} \right) + \left(235 - 92 \right) 1.008665 \text{ amu} \right] - 235.043924 \text{ amu}$$

$$\Delta m = 1.91517 \text{ amu}$$

Step 2: Use the mass defect and Equation (1-2) to calculate the binding energy.

$$B.E. = \Delta m \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right)$$

$$= 1.91517 \text{ amu} \left(\frac{931.5 \text{ MeV}}{1 \text{ amu}} \right)$$

$$= 1784 \text{ MeV}$$

Energy Levels of Atoms

The electrons that circle the nucleus move in fairly well-defined orbits. Some of these electrons are more tightly bound in the atom than others. For example, only 7.38 eV is required to remove the outermost electron from a lead atom, while 88,000 eV is required to remove the innermost electron. The process of removing an electron from an atom is called ionization, and the energy required to remove the electron is called the ionization energy.

In a neutral atom (number of electrons = Z) it is possible for the electrons to be in a variety of different orbits, each with a different energy level. The state of lowest energy is the one in which the atom is normally found and is called the ground state. When the atom possesses more energy than its ground state energy, it is said to be in an excited state.

An atom cannot stay in the excited state for an indefinite period of time. An excited atom will eventually transition to either a lower-energy excited state, or directly to its ground state, by emitting a discrete bundle of electromagnetic energy called an x-ray. The energy of the x-ray will be equal to the difference between the energy levels of the atom and will typically range from several eV to 100,000 eV in magnitude.

Energy Levels of the Nucleus

The nucleons in the nucleus of an atom, like the electrons that circle the nucleus, exist in shells that correspond to energy states. The energy shells of the nucleus are less defined and less understood than those of the electrons. There is a state of lowest energy (the ground state) and discrete possible excited states for a nucleus. Where the discrete energy states for the electrons of an atom are measured in eV or keV, the energy levels of the nucleus are considerably greater and typically measured in MeV.

A nucleus that is in the excited state will not remain at that energy level for an indefinite period. Like the electrons in an excited atom, the nucleons in an excited nucleus will transition towards their lowest energy configuration and in doing so emit a discrete bundle of electromagnetic radiation called a gamma ray (γ-ray). The only differences between x-rays and γ-rays are their energy levels and whether they are emitted from the electron shell or from the nucleus.

The ground state and the excited states of a nucleus can be depicted in a nuclear energy-level diagram. The nuclear energy-level diagram consists of a stack of horizontal bars, one bar for each of the excited states of the nucleus. The vertical distance between the bar representing an excited state and the bar representing the ground state is proportional to the energy level of the excited state with respect to the ground state. This difference in energy between the ground state and the excited state is called the excitation energy of the excited state. The ground state of a nuclide has zero excitation energy. The bars for the excited states are labeled with their respective energy levels. Figure 7 is the energy level diagram for nickel-60.

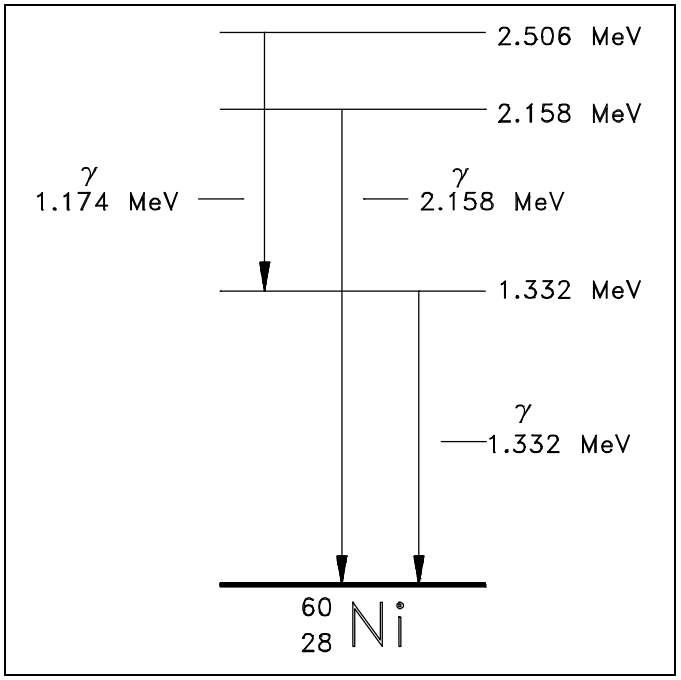


Figure 7 Energy Level Diagram - Nickel-60

Summary

The important information in this chapter is summarized below.

Mass Defect and Binding Energy Summary

- Mass defect is the difference between the mass of the atom and the sum of the masses of its constituent parts.
- Binding energy is the amount of energy that must be supplied to a nucleus to completely separate its nuclear particles. Binding energy is the energy equivalent of the mass defect.
- Mass defect can be calculated by using the equation below.

$$\Delta m = [\ Z(m_p + m_e) + (A\text{-}Z)m_n\ ] - m_{atom}$$

- Binding energy can be calculated by multiplying the mass defect by the factor of 931.5 MeV per amu.

MODES OF RADIOACTIVE DECAY

Most atoms found in nature are stable and do not emit particles or energy that change form over time. Some atoms, however, do not have stable nuclei. These atoms emit radiation in order to achieve a more stable configuration.

EO 2.1 DESCRIBE the following processes:

a. Alpha decay
b. Beta-minus decay
c. Beta-plus decay
d. Electron capture
e. Internal conversions
f. Isomeric transitions

EO 2.2 Given a Chart of the Nuclides, WRITE the radioactive decay chain for a nuclide.

EO 2.3 EXPLAIN why one or more gamma rays typically accompany particle emission.

EO 2.4 Given the stability curve on the Chart of the Nuclides, DETERMINE the type of radioactive decay that the nuclides in each region of the chart will typically undergo.

Stability of Nuclei

As mass numbers become larger, the ratio of neutrons to protons in the nucleus becomes larger for the stable nuclei. Non-stable nuclei may have an excess or deficiency of neutrons and undergo a transformation process known as beta (β) decay. Non-stable nuclei can also undergo a variety of other processes such as alpha (α) or neutron (n) decay. As a result of these decay processes, the final nucleus is in a more stable or more tightly bound configuration.

Natural Radioactivity

In 1896, the French physicist Becquerel discovered that crystals of a uranium salt emitted rays that were similar to x-rays in that they were highly penetrating, could affect a photographic plate, and induced electrical conductivity in gases. Becquerel's discovery was followed in 1898 by the identification of two other radioactive elements, polonium and radium, by Pierre and Marie Curie.

Heavy elements, such as uranium or thorium, and their unstable decay chain elements emit radiation in their naturally occurring state. Uranium and thorium, present since their creation at the beginning of geological time, have an extremely slow rate of decay. All naturally occurring nuclides with atomic numbers greater than 82 are radioactive.

Nuclear Decay

Whenever a nucleus can attain a more stable (i.e., more tightly bound) configuration by emitting radiation, a spontaneous disintegration process known as radioactive decay or nuclear decay may occur. In practice, this "radiation" may be electromagnetic radiation, particles, or both.

Detailed studies of radioactive decay and nuclear reaction processes have led to the formulation of useful conservation principles. The four principles of most interest in this module are discussed below.

Conservation of electric charge implies that charges are neither created nor destroyed. Single positive and negative charges may, however, neutralize each other. It is also possible for a neutral particle to produce one charge of each sign.

Conservation of mass number does not allow a net change in the number of nucleons. However, the conversion of a proton to a neutron and vice versa is allowed.

Conservation of mass and energy implies that the total of the kinetic energy and the energy equivalent of the mass in a system must be conserved in all decays and reactions. Mass can be converted to energy and energy can be converted to mass, but the sum of mass and energy must be constant.

Conservation of momentum is responsible for the distribution of the available kinetic energy among product nuclei, particles, and/or radiation. The total amount is the same before and after the reaction even though it may be distributed differently among entirely different nuclides and/or particles.

Alpha Decay (α)

Alpha decay is the emission of alpha particles (helium nuclei) which may be represented as either ${}^{4}_{2}\mathrm{He}$ or ${}^{4}_{2}\alpha$. When an unstable nucleus ejects an alpha particle, the atomic number is reduced by 2 and the mass number decreased by 4. An example is uranium-234 which decays by the ejection of an alpha particle accompanied by the emission of a 0.068 MeV gamma.

$$^{234}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + {}^{4}_{2}\alpha + \gamma + \mathrm{KE}$$

The combined kinetic energy of the daughter nucleus (Thorium-230) and the α particle is designated as KE. The sum of the KE and the gamma energy is equal to the difference in mass between the original nucleus (Uranium-234) and the final particles (equivalent to the binding energy released, since $\Delta m = BE$). The alpha particle will carry off as much as 98% of the kinetic energy and, in most cases, can be considered to carry off all the kinetic energy.

Beta Decay (β)

Beta decay is the emission of electrons of nuclear rather than orbital origin. These particles are electrons that have been expelled by excited nuclei and may have a charge of either sign.

If both energy and momentum are to be conserved, a third type of particle, the neutrino, ν, must be involved. The neutrino is associated with positive electron emission, and its antiparticle, the antineutrino, $\bar{\nu}$, is emitted with a negative electron. These uncharged particles have only the weakest interaction with matter, no mass, and travel at the speed of light. For all practical purposes, they pass through all materials with so few interactions that the energy they possess cannot be recovered. The neutrinos and antineutrinos are included here only because they carry a portion of the kinetic energy that would otherwise belong to the beta particle, and therefore, must be considered for energy and momentum to be conserved. They are normally ignored since they are not significant in the context of nuclear reactor applications.

Negative electron emission, represented as ${}^{0}_{-1}\mathrm{e}$, ${}^{0}_{-1}\beta$, or simply as e^- or β^-, effectively converts a neutron to a proton, thus increasing the atomic number by one and leaving the mass number unchanged. This is a common mode of decay for nuclei with an excess of neutrons, such as fission fragments below and to the right of the neutron-proton stability curve (refer to Figure 6). An example of a typical beta minus-decay reaction is shown below.

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$$

Positively charged electrons (beta-plus) are known as positrons. Except for sign, they are nearly identical to their negatively charged cousins. When a positron, represented as ${}^{0}_{+1}e$, ${}^{0}_{+1}\beta$, or simply as e^+ or β^+, is ejected from the nucleus, the atomic number is decreased by one and the mass number remains unchanged. A proton has been converted to a neutron. An example of a typical positron (beta-plus) decay is shown below.

$$ {}^{13}_{7}N \rightarrow {}^{13}_{6}C + {}^{0}_{+1}\beta + {}^{0}_{0}\nu $$

Electron Capture (EC, K-capture)

Nuclei having an excess of protons may capture an electron from one of the inner orbits which immediately combines with a proton in the nucleus to form a neutron. This process is called *electron capture* (EC). The electron is normally captured from the innermost orbit (the K-shell), and, consequently, this process is sometimes called K-capture. The following example depicts electron capture.

$$ {}^{7}_{4}Be + {}^{0}_{-1}e \rightarrow {}^{7}_{3}Li + {}^{0}_{0}\nu $$

A neutrino is formed at the same time that the neutron is formed, and energy carried off by it serves to conserve momentum. Any energy that is available due to the atomic mass of the product being appreciably less than that of the parent will appear as gamma radiation. Also, there will always be characteristic x-rays given off when an electron from one of the higher energy shells moves in to fill the vacancy in the K-shell. Electron capture is shown graphically in Figure 8.

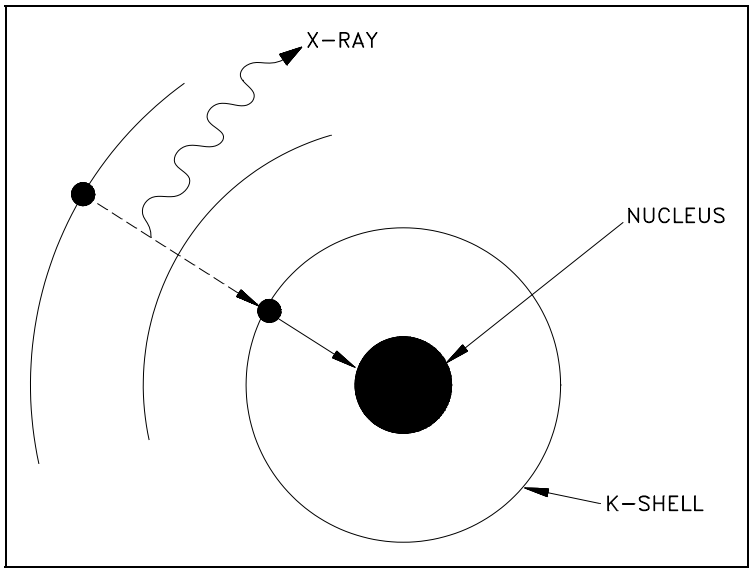


Figure 9 Orbital Electron Capture

Electron capture and positron emission result in the production of the same daughter product, and they exist as competing processes. For positron emission to occur, however, the mass of the daughter product must be less than the mass of the parent by an amount equal to at least twice the mass of an electron. This mass difference between the parent and daughter is necessary to account for two items present in the parent but not in the daughter. One item is the positron ejected from the nucleus of the parent. The other item is that the daughter product has one less orbital electron than the parent. If this requirement is not met, then orbital electron capture takes place exclusively.

Gamma Emission (γ)

Gamma radiation is a high-energy electromagnetic radiation that originates in the nucleus. It is emitted in the form of photons, discrete bundles of energy that have both wave and particle properties. Often a daughter nuclide is left in an excited state after a radioactive parent nucleus undergoes a transformation by alpha decay, beta decay, or electron capture. The nucleus will drop to the ground state by the emission of gamma radiation.

Internal Conversion

The usual method for an excited nucleus to go from the excited state to the ground state is by emission of gamma radiation. However, in some cases the gamma ray (photon) emerges from the nucleus only to interact with one of the innermost orbital electrons and, as a result, the energy of the photon is transferred to the electron. The gamma ray is then said to have undergone *internal conversion*. The conversion electron is ejected from the atom with kinetic energy equal to the gamma energy minus the binding energy of the orbital electron. An orbital electron then drops to a lower energy state to fill the vacancy, and this is accompanied by the emission of characteristic x-rays.

Isomers and Isomeric Transition

Isomeric transition commonly occurs immediately after particle emission; however, the nucleus may remain in an excited state for a measurable period of time before dropping to the ground state at its own characteristic rate. A nucleus that remains in such an excited state is known as a nuclear *isomer* because it differs in energy and behavior from other nuclei with the same atomic number and mass number. The decay of an excited nuclear isomer to a lower energy level is called an *isomeric transition*. It is also possible for the excited isomer to decay by some alternate means, for example, by beta emission.

An example of gamma emission accompanying particle emission is illustrated by the decay of nitrogen-16 below.

$$^{16}_{7}\mathrm{N} \rightarrow \left(^{16}_{8}\mathrm{O}\right)^{*} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$$

$$\left(^{16}_{8}\mathrm{O}\right)^{*} \rightarrow {}^{16}_{8}\mathrm{O} + {}^{0}_{0}\gamma$$

Decay Chains

When an unstable nucleus decays, the resulting daughter nucleus is not necessarily stable. The nucleus resulting from the decay of a parent is often itself unstable, and will undergo an additional decay. This is especially common among the larger nuclides.

It is possible to trace the steps of an unstable atom as it goes through multiple decays trying to achieve stability. The list of the original unstable nuclide, the nuclides that are involved as intermediate steps in the decay, and the final stable nuclide is known as the *decay chain*. One common method for stating the decay chain is to state each of the nuclides involved in the standard ${}^{A}_{Z}X$ format. Arrows are used between nuclides to indicate where decays occur, with the type of decay indicated above the arrow and the half-life below the arrow. The half-life for decay will be discussed in the next chapter.

Example:

> Write the decay chains for rubidium-91 and actinium-215. Continue the chains until a stable nuclide or a nuclide with a half-life greater than 1 x 10^6 years is reached.

Solution:

$$^{91}_{37}\mathrm{Rb} \xrightarrow[58.0\ \mathrm{s}]{\beta^-} {}^{91}_{38}\mathrm{Sr} \xrightarrow[9.5\ \mathrm{hrs}]{\beta^-} {}^{91}_{39}\mathrm{Y} \xrightarrow[58.5\ \mathrm{d}]{\beta^-} {}^{91}_{40}\mathrm{Zr}$$

$$^{215}_{85}\mathrm{At} \xrightarrow[0.10\ \mathrm{ms}]{\alpha} {}^{211}_{83}\mathrm{Bi} \xrightarrow[2.14\ \mathrm{min}]{\alpha} {}^{207}_{81}\mathrm{Tl} \xrightarrow[4.77\ \mathrm{min}]{\beta^-} {}^{207}_{82}\mathrm{Pb}$$

Predicting Type of Decay

Radioactive nuclides tend to decay in a way that results in a daughter nuclide that lies closer to the line of stability. Due to this, it is possible to predict the type of decay that a nuclide will undergo based on its location relative to the line of stability on the Chart of the Nuclides.

Figure 9 illustrates the type of decay nuclides in different regions of the chart will typically undergo. Nuclides that are below and to the right of the line of stability will usually undergo β^- decay. Nuclides that are above and to the left of the line of stability will usually undergo either β^+ decay or electron capture. Most nuclides that will undergo α decay are found in the upper right hand region of the chart. These are general rules that have many exceptions, especially in the region of the heavy nuclides.

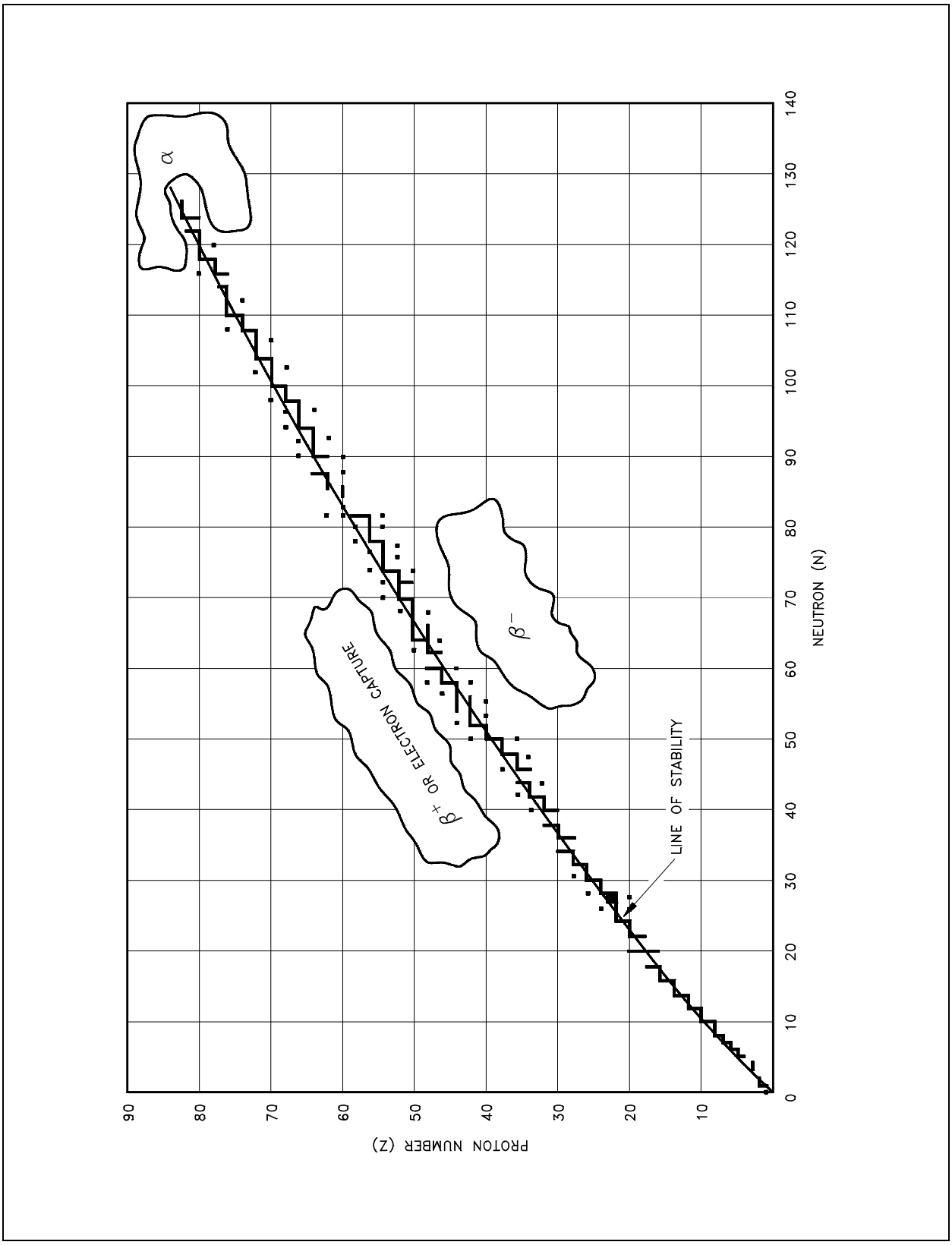


Figure 9 Types of Radioactive Decay Relative to the Line of Stability

Summary

The important information in this chapter is summarized below.

Modes of Radioactive Decay Summary

- Alpha decay is the emission of an alpha particle (2 protons and 2 neutrons) from an unstable nucleus. The daughter nuclide has an atomic number 2 less than the parent nuclide and a mass number 4 less than the parent nuclide. The daughter nucleus commonly releases its excitation energy by gamma emission.
- Beta-minus decay effectively converts a neutron to a proton and an electron, which is immediately ejected from the nucleus. The daughter nuclide has its atomic number increased by 1 and the same mass number compared to the parent.
- Beta-plus decay effectively converts a proton to a neutron and a positron, which is immediately ejected from the nucleus. The daughter nuclide has its atomic number decreased by 1 and the same mass number compared to the parent.
- In electron capture, the nucleus absorbs an electron from the innermost orbit. This electron combines with a proton to form a neutron.
- Internal conversion occurs when a gamma ray, emitted by the nucleus as it goes from the excited state to the ground state, interacts with one of the innermost electrons of the same atom. The electron is ejected from the atom.
- An isomeric transition is the decay of an excited nucleus to a lower-energy level by the emission of a gamma ray.
- Decay chains can be found by tracing the steps an unstable atom goes through as it tries to achieve stability.
- Many modes of radioactive decay result in a daughter nuclide that has an energy level above the ground state. This excitation energy is usually released immediately in the form of a gamma ray.
- The type of decay that a nuclide will typically undergo can be determined by its relationship to the line of stability on the Chart of the Nuclides. Nuclides that lie below and to the right of the line of stability will typically beta minus decay. Nuclides above and to the left of the line will typically either beta plus decay or electron capture. Most alpha emitters are found in the upper, right-hand corner of the chart.

RADIOACTIVITY

The rate at which a sample of radioactive material decays is not constant. As individual atoms of the material decay, there are fewer of those types of atoms remaining. Since the rate of decay is directly proportional to the number of atoms, the rate of decay will decrease as the number of atoms decreases.

EO 2.5 DEFINE the following terms:

a. Radioactivity
b. Curie
c. Becquerel
d. Radioactive decay constant
e. Radioactive half-life

EO 2.6 Given the number of atoms and either the half-life or decay constant of a nuclide, CALCULATE the activity.

EO 2.7 Given the initial activity and the decay constant of a nuclide, CALCULATE the activity at any later time.

EO 2.8 CONVERT between the half-life and decay constant for a nuclide.

EO 2.9 Given the Chart of the Nuclides and the original activity, PLOT the radioactive decay curve for a nuclide on either linear or semi-log coordinates.

EO 2.10 DEFINE the following terms:

a. Radioactive equilibrium
b. Transient radioactive equilibrium

Radioactive Decay Rates

Radioactivity is the property of certain nuclides of spontaneously emitting particles or gamma radiation. The decay of radioactive nuclides occurs in a random manner, and the precise time at which a single nucleus will decay cannot be determined. However, the average behavior of a very large sample can be predicted accurately by using statistical methods. These studies have revealed that there is a certain probability that in a given time interval a certain fraction of the nuclei within a sample of a particular nuclide will decay. This probability per unit time that an atom of a nuclide will decay is known as the *radioactive decay constant*, λ. The units for the decay constant are inverse time such as 1/second, 1/minute, 1/hour, or 1/year. These decay constant units can also be expressed as $second^{-1}$, $minute^{-1}$, $hour^{-1}$, and $year^{-1}$.

The *activity* (A) of a sample is the rate of decay of that sample. This rate of decay is usually measured in the number of disintegrations that occur per second. For a sample containing millions of atoms, the activity is the product of the decay constant and the number of atoms present in the sample.

The relationship between the activity, number of atoms, and decay constant is shown in Equation (1-3).

$$A = \lambda N \qquad (1\text{-}3)$$

where:

A = Activity of the nuclide (disintegrations/second)
λ = decay constant of the nuclide (second^{-1})
N = Number of atoms of the nuclide in the sample

Since λ is a constant, the activity and the number of atoms are always proportional.

Units of Measurement for Radioactivity

Two common units to measure the activity of a substance are the curie (Ci) and becquerel (Bq). A *curie* is a unit of measure of the rate of radioactive decay equal to 3.7×10^{10} disintegrations per second. This is approximately equivalent to the number of disintegrations that one gram of radium-226 will undergo in one second. A *becquerel* is a more fundamental unit of measure of radioactive decay that is equal to 1 disintegration per second. Currently, the curie is more widely used in the United States, but usage of the becquerel can be expected to broaden as the metric system slowly comes into wider use. The conversion between curies and becquerels is shown below.

1 curie = 3.7×10^{10} becquerels

Variation of Radioactivity Over Time

The rate at which a given radionuclide sample decays is stated in Equation (1-3) as being equal to the product of the number of atoms and the decay constant. From this basic relationship it is possible to use calculus to derive an expression which can be used to calculate how the number of atoms present will change over time. The derivation is beyond the scope of this text, but Equation (1-4) is the useful result.

$$N = N_o e^{-\lambda t} \qquad (1\text{-}4)$$

where:

N = number of atoms present at time t
N_o = number of atoms initially present
λ = decay constant (time^{-1})
t = time

Since the activity and the number of atoms are always proportional, they may be used interchangeably to describe any given radionuclide population. Therefore, the following is true.

$$A = A_o\, e^{-\lambda t} \tag{1-5}$$

where:

A = activity present at time t
A_o = activity initially present
λ = decay constant (time^{-1})
t = time

Radioactive Half-Life

One of the most useful terms for estimating how quickly a nuclide will decay is the radioactive half-life. The *radioactive half-life* is defined as the amount of time required for the activity to decrease to one-half of its original value. A relationship between the half-life and decay constant can be developed from Equation (1-5). The half-life can be calculated by solving Equation (1-5) for the time, t, when the current activity, A, equals one-half the initial activity A_o.

First, solve Equation (1-5) for t.

$$A = A_o\, e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

If A is equal to one-half of A_o, then A/A_o is equal to one-half. Substituting this in the equation above yields an expression for $t_{1/2}$.

$$t_{1/2} = \frac{-\ln\left(\frac{1}{2}\right)}{\lambda} \tag{1-6}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

The basic features of decay of a radionuclide sample are shown by the graph in Figure 10.

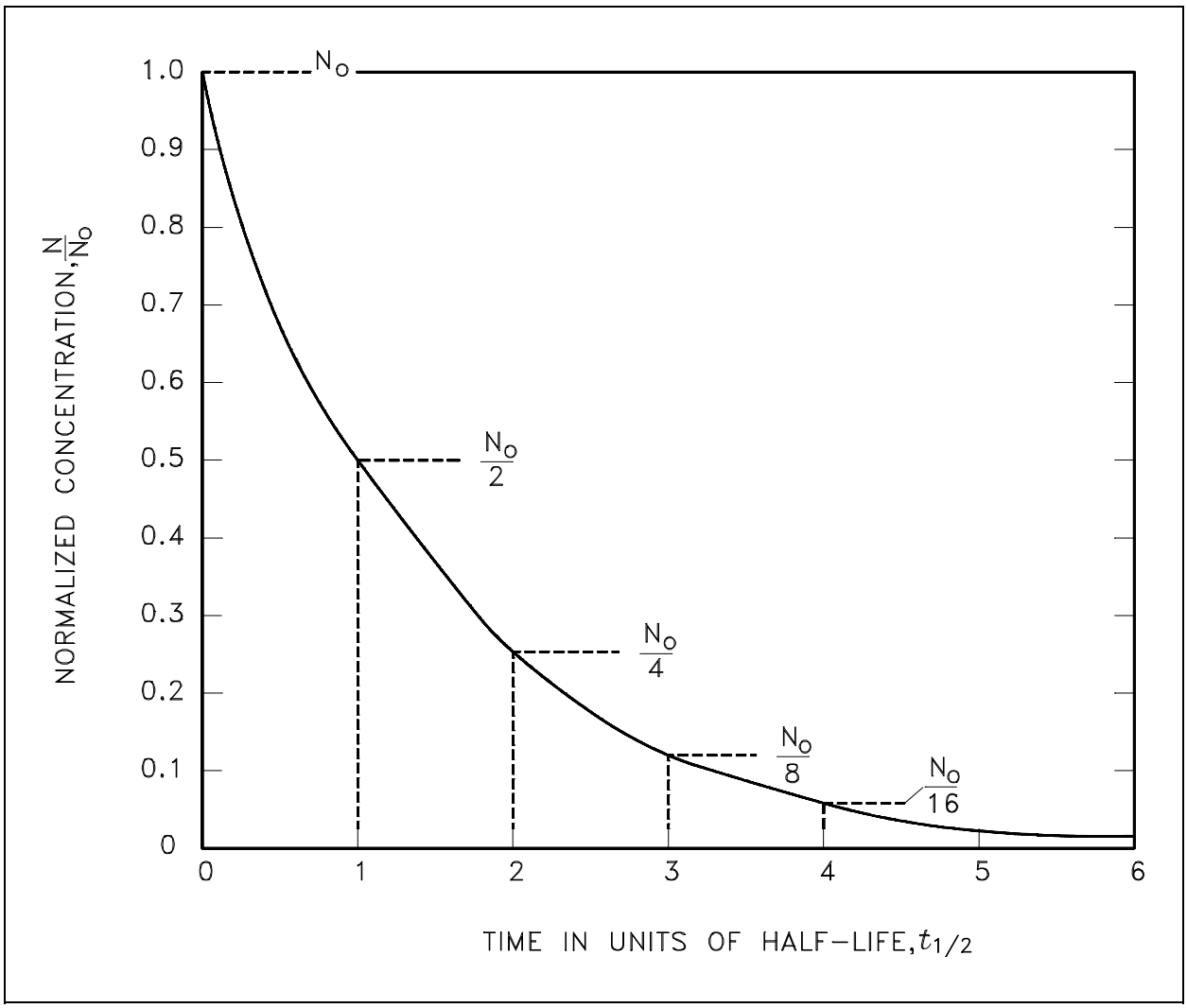


Figure 10 Radioactive Decay as a Function of Time in Units of Half-Life

Assuming an initial number of atoms N_o, the population, and consequently, the activity may be noted to decrease by one-half of this value in a time of one half-life. Additional decreases occur so that whenever one half-life elapses, the number of atoms drops to one-half of what its value was at the beginning of that time interval. After five half-lives have elapsed, only 1/32, or 3.1%, of the original number of atoms remains. After seven half-lives, only 1/128, or 0.78%, of the atoms remains. The number of atoms existing after 5 to 7 half-lives can usually be assumed to be negligible. The Chemistry Fundamentals Handbook contains additional information on calculating the number of atoms contained within a sample.

Example:

A sample of material contains 20 micrograms of californium-252.

Californium-252 has a half-life of 2.638 years.

Calculate:

(a) The number of californium-252 atoms initially present
(b) The activity of the californium-252 in curies
(c) The number of californium-252 atoms that will remain in 12 years
(d) The time it will take for the activity to reach 0.001 curies

Solution:

(a) The number of atoms of californium-252 can be determined as below.

$$N_{Cf-252} = \text{mass}\left(\frac{1\ \text{mole}}{\text{isotopic mass}}\right)\left(\frac{N_A}{1\ \text{mole}}\right)$$

$$= (20 \times 10^{-6}\ \text{g})\left(\frac{1\ \text{mole}}{252.08\ \text{g}}\right)\left(\frac{6.022 \times 10^{23}\ \text{atoms}}{1\ \text{mole}}\right)$$

$$= 4.78 \times 10^{16}\ \text{atoms}$$

(b) First, use Equation (1-6) to calculate the decay constant.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{2.638\ \text{years}}$$

$$= 0.263\ \text{year}^{-1}$$

Use this value for the decay constant in Equation (1-3) to determine the activity.

$$A = \lambda N$$

$$= (0.263\ \text{year}^{-1})(4.78 \times 10^{16}\ \text{atoms})\left(\frac{1\ \text{year}}{365.25\ \text{days}}\right)\left(\frac{1\ \text{day}}{24\ \text{hours}}\right)\left(\frac{1\ \text{hour}}{3600\ \text{seconds}}\right)$$

$$= \left(3.98 \times 10^{8}\ \frac{\text{disintegrations}}{\text{second}}\right)\left(\frac{1\ \text{curie}}{3.7 \times 10^{10}\ \text{disintegrations/sec}}\right)$$

$$= 0.0108\ \text{curies}$$

(c) The number of californium atoms that will remain in 12 years can be calculated from Equation (1-4).

$$N = N_o\, e^{-\lambda t}$$

$$= \left(4.78 \times 10^{16}\right) e^{-(0.263/\text{yr})\,(12\ \text{yr})}$$

$$= 2.04 \times 10^{15}$$

(d) The time that it will take for the activity to reach 0.001 Ci can be determined from Equation (1-5). First, solve Equation (1-5) for time.

$$A = A_o\, e^{-\lambda t}$$

$$\frac{A}{A_o} = e^{-\lambda t}$$

$$\ln\left(\frac{A}{A_o}\right) = -\lambda t$$

$$t = \frac{-\ln\left(\frac{A}{A_o}\right)}{\lambda}$$

Inserting the appropriate values in the right side of this equation will result in the required time.

$$t = \frac{-\ln\left(\frac{0.001\ \text{Ci}}{0.0108\ \text{Ci}}\right)}{0.263\ \text{year}^{-1}}$$

$$t = 9.05\ \text{years}$$

Plotting Radioactive Decay

It is useful to plot the activity of a nuclide as it changes over time. Plots of this type can be used to determine when the activity will fall below a certain level. This plot is usually done showing activity on either a linear or a logarithmic scale. The decay of the activity of a single nuclide on a logarithmic scale will plot as a straight line because the decay is exponential.

Example:

Plot the radioactive decay curve for nitrogen-16 over a period of 100 seconds. The initial activity is 142 curies and the half-life of nitrogen-16 is 7.13 seconds. Plot the curve on both linear rectangular coordinates and on a semi-log scale.

Solution:

First, use Equation (1-6) to calculate the decay constant corresponding to a half-life of 7.13 seconds.

$$t_{1/2} = \frac{0.693}{\lambda}$$

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$\lambda = \frac{0.693}{7.13 \text{ seconds}}$$

$$\lambda = 0.0972 \text{ second}^{-1}$$

Use the decay constant determined above to calculate the activity at various times using Equation (1-5).

$$A = A_o e^{-\lambda t}$$

Time	Activity
0 seconds	142 Ci
20 seconds	20.3 Ci
40 seconds	2.91 Ci
60 seconds	0.416 Ci
80 seconds	0.0596 Ci
100 seconds	0.00853 Ci

Plotting the data points calculated above on both linear and semilog scales results in the graphs shown in Figure 11.

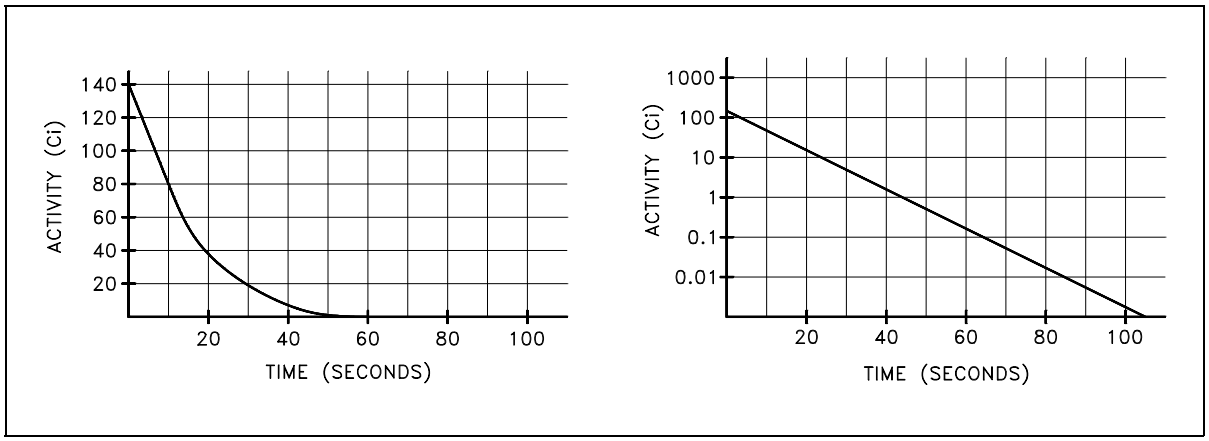


Figure 11 Linear and Semi-log Plots of Nitrogen-16 Decay

If a substance contains more than one radioactive nuclide, the total activity is the sum of the individual activities of each nuclide. As an example, consider a sample of material that contained 1×10^6 atoms of iron-59 that has a half-life of 44.51 days ($\lambda = 1.80 \times 10^{7}$ sec^{1}), 1×10^6 atoms of manganese-54 that has a half-life of 312.2 days ($\lambda = 2.57 \times 10^{-8}$ sec^{-1}), and 1×10^6 atoms of cobalt-60 that has a half-life of 1925 days ($\lambda = 4.17 \times 10^{-9}$ sec^{-1}).

The initial activity of each of the nuclides would be the product of the number of atoms and the decay constant.

$$A_{Fe-59} = N_{Fe-59}\,\lambda_{Fe-59}$$
$$= (1 \times 10^6 \text{ atoms})\,(1.80 \times 10^{-7} \text{ sec}^{-1})$$
$$= 0.180 \text{ Ci}$$

$$A_{Mn-54} = N_{Mn-54}\,\lambda_{Mn-54}$$
$$= (1 \times 10^6 \text{ atoms})\,(2.57 \times 10^{-8} \text{ sec}^{-1})$$
$$= 0.0257 \text{ Ci}$$

$$A_{Co-60} = N_{Co-60}\,\lambda_{Co-60}$$
$$= (1 \times 10^6 \text{ atoms})\,(4.17 \times 10^{-9} \text{ sec}^{-1})$$
$$= 0.00417 \text{ Ci}$$

Plotting the manner in which the activities of each of the three nuclides decay over time demonstrates that initially the activity of the shortest-lived nuclide (iron-59) dominates the total activity, then manganese-54 dominates. After almost all of the iron and manganese have decayed away, the only contributor to activity will be the cobalt-60. A plot of this combined decay is shown in Figure 12.

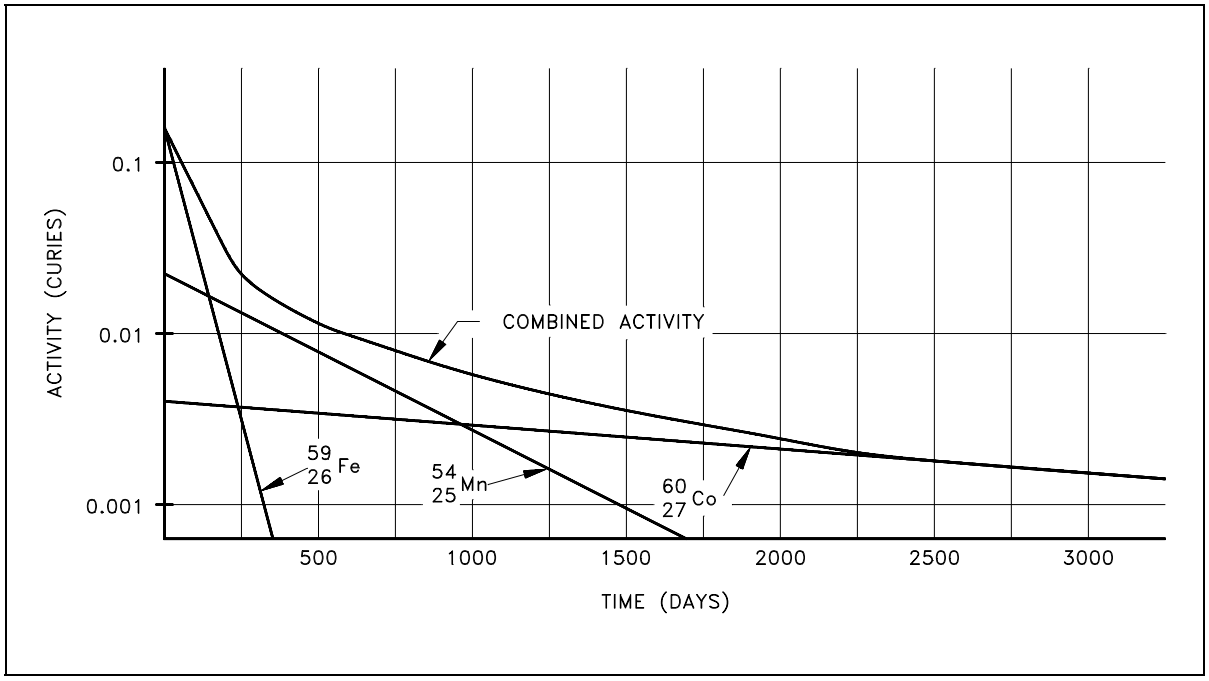


Figure 12 Combined Decay of Iron-56, Manganese-54, and Cobalt-60

Radioactive Equilibrium

Radioactive equilibrium exists when a radioactive nuclide is decaying at the same rate at which it is being produced. Since the production rate and decay rate are equal, the number of atoms present remains constant over time.

An example of radioactive equilibrium is the concentration of sodium-24 in the coolant circulating through a sodium-cooled nuclear reactor. Assume that the sodium-24 is being produced at a rate of 1 x 10^6 atoms per second. If the sodium-24 were stable and did not decay, the amount of sodium-24 present after some period of time could be calculated by multiplying the production rate by the amount of time. Plotting the amount of material present would result in the graph in Figure 13.

However, sodium-24 is not stable, and it decays with a half-life of 14.96 hours. If no sodium-24 is present initially and production starts at a rate of 1 x 10^6 atoms per second, the rate of decay will initially be zero because there is no sodium-24 present to decay. The rate of decay of sodium-24 will increase as the amount of sodium-24 increases.

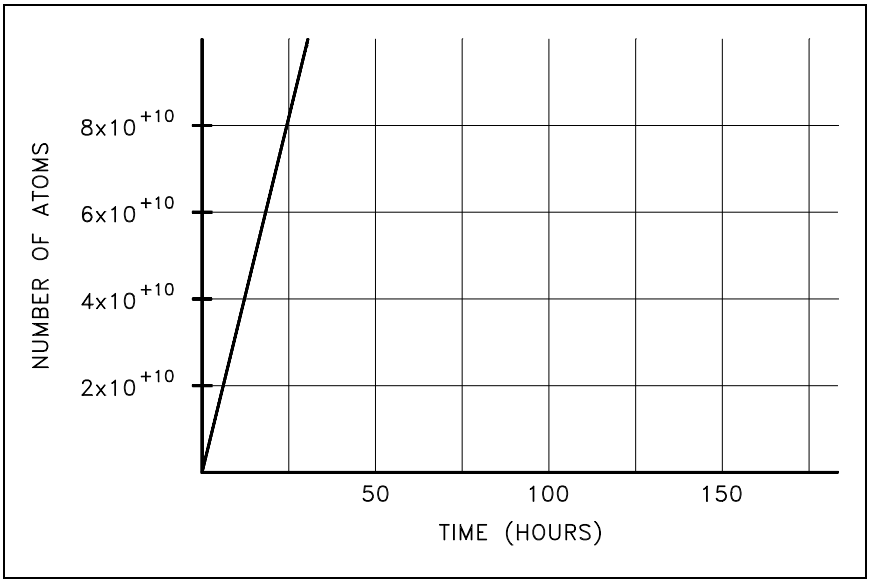


Figure 13 Cumulative Production of Sodium-24 Over Time

The amount of sodium-24 present will initially increase rapidly, then it will increase at a continually decreasing rate until the rate of decay is equal to the rate of production. It is possible to calculate how much sodium-24 will be present at equilibrium by setting the production rate (R) equal to the decay rate (λ N).

$$\mathbf{R} = \lambda \mathbf{N}$$

$$\mathbf{N} = \frac{\mathbf{R}}{\lambda}$$

where:

R = production rate (atoms/second)
λ = decay constant (second^{-1})
N = number of atoms

It is possible to calculate the equilibrium value for sodium-24 being produced at a rate of 1 x 10^6 atoms/second.

$$\lambda = \frac{0.693}{t_{1/2}}$$

$$= \frac{0.693}{14.96 \text{ hours}} \left(\frac{1 \text{ hour}}{3600 \text{ seconds}} \right)$$

$$= 1.287 \text{ x } 10^{-5} \text{ second}^{-1}$$

$$\mathbf{N} = \frac{\mathbf{R}}{\lambda}$$

$$= \frac{1 \text{ x } 10^6 \frac{\text{atoms}}{\text{second}}}{1.287 \text{ x } 10^{-5} \text{ second}^{-1}}$$

$$= 7.77 \text{ x } 10^{10} \text{ atoms}$$

The development of the equation to calculate how the amount of sodium-24 changes over time as it approaches the equilibrium value is beyond the scope of this handbook. However, the equation is presented below.

$$N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$$

This equation can be used to calculate the values of the amount of sodium-24 present at different times. As the time increases, the exponential term approaches zero, and the number of atoms present will approach R/λ. A plot of the approach of sodium-24 to equilibrium is shown in Figure 14.

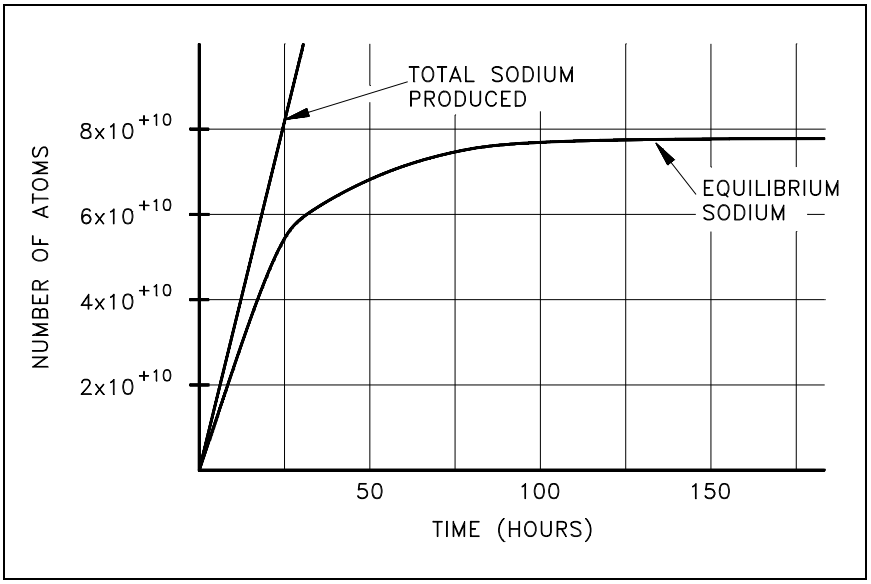


Figure 14 Approach of Sodium-24 to Equilibrium

Transient Radioactive Equilibrium

Transient radioactive equilibrium occurs when the parent nuclide and the daughter nuclide decay at essentially the same rate.

For transient equilibrium to occur, the parent must have a long half-life when compared to the daughter. An example of this type of compound decay process is barium-140, which decays by beta emission to lanthanum-140, which in turn decays by beta emission to stable cerium-140.

$$^{140}_{56}\mathrm{Ba} \xrightarrow[12.75\ \text{days}]{\beta^-} {}^{140}_{57}\mathrm{La} \xrightarrow[1.678\ \text{days}]{\beta^-} {}^{140}_{58}\mathrm{Ce}$$

The decay constant for barium-140 is considerably smaller than the decay constant for lanthanum-140. Remember that the rate of decay of both the parent and daughter can be represented as λN. Although the decay constant for barium-140 is smaller, the actual rate of decay (λN) is initially larger than that of lanthanum-140 because of the great difference in their initial concentrations. As the concentration of the daughter increases, the rate of decay of the daughter will approach and eventually match the decay rate of the parent. When this occurs, they are said to be in transient equilibrium. A plot of the barium-lanthanum-cerium decay chain reaching transient equilibrium is shown in Figure 15.

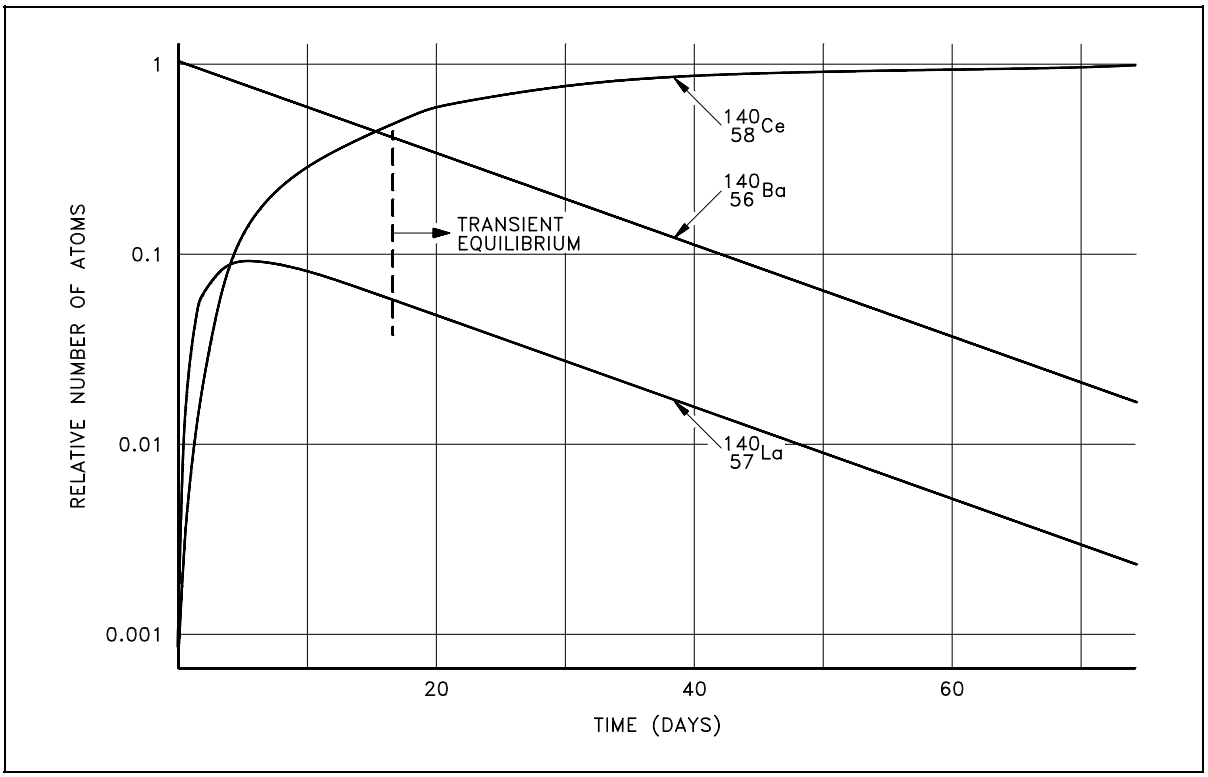


Figure 15 Transient Equilibrium in the Decay of Barium-140

Secular equilibrium occurs when the parent has an extremely long half-life. In the long decay chain for a naturally radioactive element, such as thorium-232, where all of the elements in the chain are in secular equilibrium, each of the descendants has built up to an equilibrium amount and all decay at the rate set by the original parent. The only exception is the final stable element on the end of the chain. Its number of atoms is constantly increasing.

Summary

The important information in this chapter is summarized on the following page.

Radioactivity Summary

- Radioactivity is the decay of unstable atoms by the emission of particles and electromagnetic radiation.
- A curie (Ci) is a unit of radioactivity equal to 3.7×10^{10} disintegrations per second.
- A becquerel (Bq) is a unit of radioactivity equal to 1 disintegration per second.
- The radioactive decay constant (λ) is the probability per unit time that an atom will decay.
- The radioactive half-life is the amount of time required for the activity to decrease to one-half its original value.
- The activity of a substance can be calculated from the number of atoms and the decay constant based on the equation below.

$$A = \lambda N$$

- The amount of activity remaining after a particular time can be calculated from the equation below.

$$A = A_o e^{-\lambda t}$$

- The relationship between the decay constant and the half-life is shown below.

$$t_{1/2} = \frac{0.693}{\lambda}$$

- Plots of radioactive decay can be useful to describe the variation of activity over time. If decay is plotted using semi-log scale the plot results in a straight line.
- Radioactive equilibrium exists when the production rate of a material is equal to the removal rate.
- Transient radioactive equilibrium exists when the parent nuclide and the daughter nuclide decay at essentially the same rate. This occurs only when the parent has a long half-life compared to the daughter.

NEUTRON INTERACTIONS

Neutrons can cause many different types of interactions. The neutron may simply scatter off the nucleus in two different ways, or it may actually be absorbed into the nucleus. If a neutron is absorbed into the nucleus, it may result in the emission of a gamma ray or a subatomic particle, or it may cause the nucleus to fission.

EO 3.1 DESCRIBE the following scattering interactions between a neutron and a nucleus:

a. Elastic scattering
b. Inelastic scattering

EO 3.2 STATE the conservation laws that apply to an elastic collision between a neutron and a nucleus.

EO 3.3 DESCRIBE the following reactions where a neutron is absorbed in a nucleus:

a. Radiative capture
b. Particle ejection

Scattering

A neutron *scattering* reaction occurs when a nucleus, after having been struck by a neutron, emits a single neutron. Despite the fact that the initial and final neutrons do not need to be (and often are not) the same, the net effect of the reaction is as if the projectile neutron had merely "bounced off," or scattered from, the nucleus. The two categories of scattering reactions, elastic and inelastic scattering, are described in the following paragraphs.

Elastic Scattering

In an *elastic scattering* reaction between a neutron and a target nucleus, there is no energy transferred into nuclear excitation. Momentum and kinetic energy of the "system" are conserved although there is usually some transfer of kinetic energy from the neutron to the target nucleus. The target nucleus gains the amount of kinetic energy that the neutron loses.

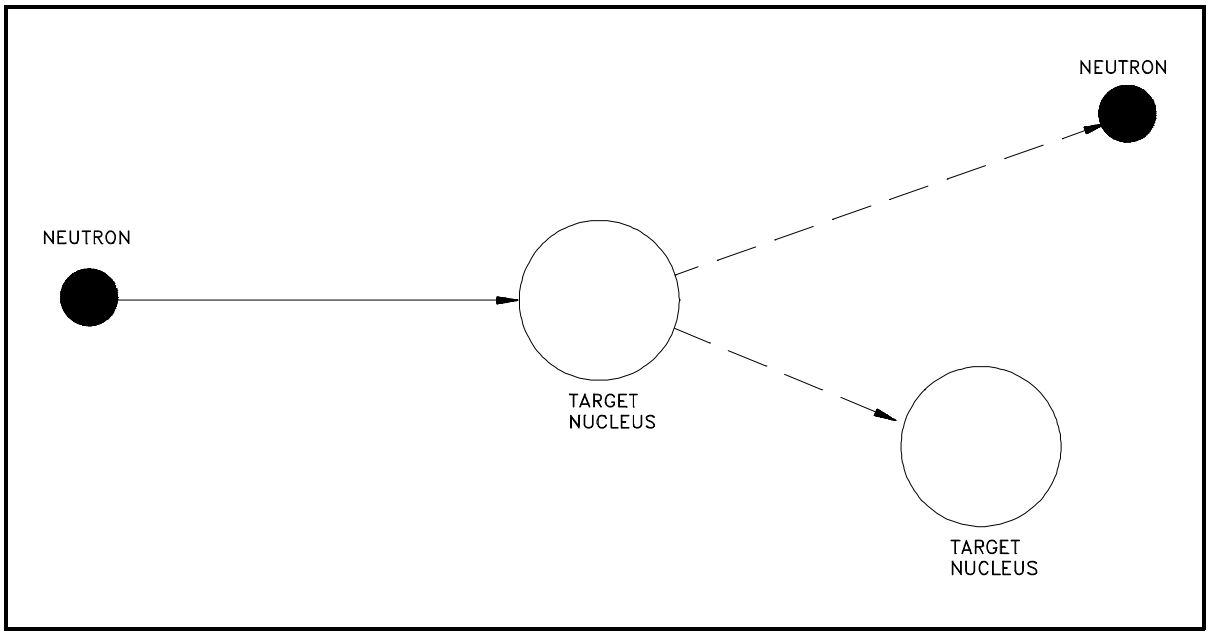


Figure 16 Elastic Scattering

Figure 16 illustrates the process of elastic scattering of a neutron off a target nucleus. In the elastic scattering reaction, the conservation of momentum and kinetic energy is represented by the equations below.

Conservation of momentum (mv)

$$(m_n\ v_{n,i}) + (m_T\ v_{T,i}) = (m_n\ v_{n,f}) + (m_T\ v_{T,f})$$

Conservation of kinetic energy $\left(\frac{1}{2}\ m\ v^2\right)$

$$\left(\frac{1}{2}\ m_n\ v_{n,i}^2\right) + \left(\frac{1}{2}\ m_T\ v_{T,i}^2\right) = \left(\frac{1}{2}\ m_n\ v_{n,f}^2\right) + \left(\frac{1}{2}\ m_T\ v_{T,f}^2\right)$$

where:

m_n = mass of the neutron
m_T = mass of the target nucleus
$v_{n,i}$ = initial neutron velocity
$v_{n,f}$ = final neutron velocity
$v_{T,i}$ = initial target velocity
$v_{T,f}$ = final target velocity

Elastic scattering of neutrons by nuclei can occur in two ways. The more unusual of the two interactions is the absorption of the neutron, forming a compound nucleus, followed by the re-emission of a neutron in such a way that the total kinetic energy is conserved and the nucleus returns to its ground state. This is known as *resonance elastic scattering* and is very dependent upon the initial kinetic energy possessed by the neutron. Due to formation of the compound nucleus, it is also referred to as compound elastic scattering. The second, more usual method, is termed *potential elastic scattering* and can be understood by visualizing the neutrons and nuclei to be much like billiard balls with impenetrable surfaces. Potential scattering takes place with incident neutrons that have an energy of up to about 1 MeV. In potential scattering, the neutron does not actually touch the nucleus and a compound nucleus is not formed. Instead, the neutron is acted on and scattered by the short range nuclear forces when it approaches close enough to the nucleus.

Inelastic Scattering

In *inelastic scattering*, the incident neutron is absorbed by the target nucleus, forming a compound nucleus. The compound nucleus will then emit a neutron of lower kinetic energy which leaves the original nucleus in an excited state. The nucleus will usually, by one or more gamma emissions, emit this excess energy to reach its ground state. Figure 17 shows the process of inelastic scattering.

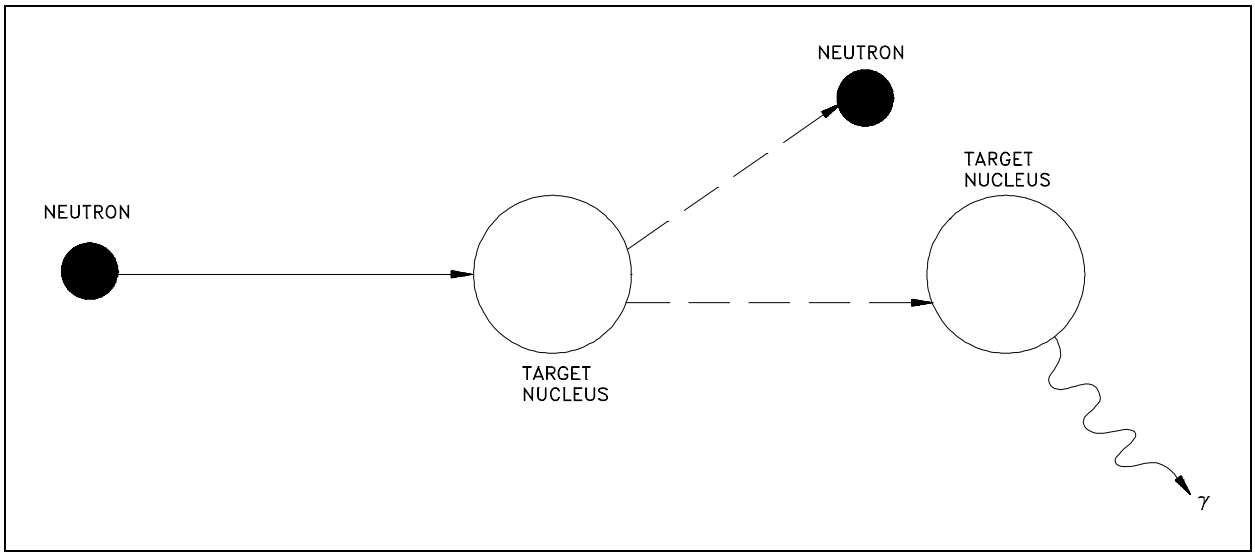


Figure 17 Inelastic Scattering

For the nucleus that has reached its ground state, the sum of the kinetic energy of the exit neutron, the target nucleus, and the total gamma energy emitted is equal to the initial kinetic energy of the incident neutron.

Absorption Reactions

Most *absorption reactions* result in the loss of a neutron coupled with the production of a charged particle or gamma ray. When the product nucleus is radioactive, additional radiation is emitted at some later time. Radiative capture, particle ejection, and fission are all categorized as absorption reactions and are briefly described below.

Radiative Capture

In *radiative capture* the incident neutron enters the target nucleus forming a compound nucleus. The compound nucleus then decays to its ground state by gamma emission. An example of a radiative capture reaction is shown below.

$$ {}^{1}_{0}n + {}^{238}_{92}U \rightarrow \left({}^{239}_{92}U\right)^{*} \rightarrow {}^{239}_{92}U + {}^{0}_{0}\gamma $$

Particle Ejection

In a *particle ejection* reaction the incident particle enters the target nucleus forming a compound nucleus. The newly formed compound nucleus has been excited to a high enough energy level to cause it to eject a new particle while the incident neutron remains in the nucleus. After the new particle is ejected, the remaining nucleus may or may not exist in an excited state depending upon the mass-energy balance of the reaction. An example of a particle ejection reaction is shown below.

$$ {}^{1}_{0}n + {}^{10}_{5}B \rightarrow \left({}^{11}_{5}B\right)^{*} \rightarrow {}^{7}_{3}Li + {}^{4}_{2}\alpha $$

Fission

One of the most important interactions that neutrons can cause is fission, in which the nucleus that absorbs the neutron actually splits into two similarly sized parts. Fission will be discussed in detail in the next chapter.

Summary

The important information in this chapter is summarized below.

Neutron Interactions Summary

- Interactions where a neutron scatters off a target nucleus are either elastic or inelastic. In elastic scattering, kinetic energy and momentum are conserved and no energy is transferred into excitation energy of the target nucleus. In inelastic scattering, some amount of kinetic energy is transferred into excitation energy of the target nucleus.
- The conservation principles that apply to an elastic collision are conservation of kinetic energy and conservation of momentum.
- Radiative capture is the absorption of a neutron by the target nucleus, resulting in an excited nucleus which subsequently (typically within a small fraction of a second) releases its excitation energy in the form of a gamma ray.
- Particle ejection occurs when a neutron is absorbed by a target nucleus, resulting in the formation of a compound nucleus. The compound nucleus immediately ejects a particle (for example, alpha or proton).

NUCLEAR FISSION

Nuclear fission is a process in which an atom splits and releases energy, fission products, and neutrons. The neutrons released by fission can, in turn, cause the fission of other atoms.

EO 4.1 **EXPLAIN the fission process using the liquid drop model of a nucleus.**

EO 4.2 **DEFINE the following terms:**

a. Excitation energy (E_{exc})
b. Critical energy (E_{crit})

EO 4.3 **DEFINE the following terms:**

a. Fissile material
b. Fissionable material
c. Fertile material

EO 4.4 **DESCRIBE the processes of transmutation, conversion, and breeding.**

EO 4.5 **DESCRIBE the curve of Binding Energy per Nucleon versus mass number and give a qualitative description of the reasons for its shape.**

EO 4.6 **EXPLAIN why only the heaviest nuclei are easily fissioned.**

EO 4.7 **EXPLAIN why uranium-235 fissions with thermal neutrons and uranium-238 fissions only with fast neutrons.**

Fission

In the fission reaction the incident neutron enters the heavy target nucleus, forming a compound nucleus that is excited to such a high energy level ($E_{exc} > E_{crit}$) that the nucleus "splits" (fissions) into two large fragments plus some neutrons. An example of a typical fission reaction is shown below.

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U \right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n \right) $$

A large amount of energy is released in the form of radiation and fragment kinetic energy.

Liquid Drop Model of a Nucleus

The nucleus is held together by the attractive nuclear force between nucleons, which was discussed in a previous chapter. The characteristics of the nuclear force are listed below.

(a) very short range, with essentially no effect beyond nuclear dimensions ($\sim 10^{-13}$ cm)

(b) stronger than the repulsive electrostatic forces within the nucleus

(c) independent of nucleon pairing, in that the attractive forces between pairs of neutrons are no different than those between pairs of protons or a neutron and a proton

(d) saturable, that is, a nucleon can attract only a few of its nearest neighbors

One theory of fission considers the fissioning of a nucleus similar in some respects to the splitting of a liquid drop. This analogy is justifiable to some extent by the fact that a liquid drop is held together by molecular forces that tend to make the drop spherical in shape and that try to resist any deformation in the same manner as nuclear forces are assumed to hold the nucleus together. By considering the nucleus as a liquid drop, the fission process can be described.

Referring to Figure 18(A), the nucleus in the ground state is undistorted, and its attractive nuclear forces are greater than the repulsive electrostatic forces between the protons within the nucleus. When an incident particle (in this instance a neutron) is absorbed by the target nucleus, a compound nucleus is formed. The compound nucleus temporarily contains all the charge and mass involved in the reaction and exists in an excited state. The excitation energy added to the compound nucleus is equal to the binding energy contributed by the incident particle plus the kinetic energy possessed by that particle. Figure 18(B) illustrates the excitation energy thus imparted to the compound nucleus, which may cause it to oscillate and become distorted. If the excitation energy is greater than a certain critical energy, the oscillations may cause the compound nucleus to become dumbbell-shaped. When this happens, the attractive nuclear forces (short-range) in the neck area are small due to saturation, while the repulsive electrostatic forces (long-range) are only slightly less than before. When the repulsive electrostatic forces exceed the attractive nuclear forces, nuclear fission occurs, as illustrated in Figure 18(C).

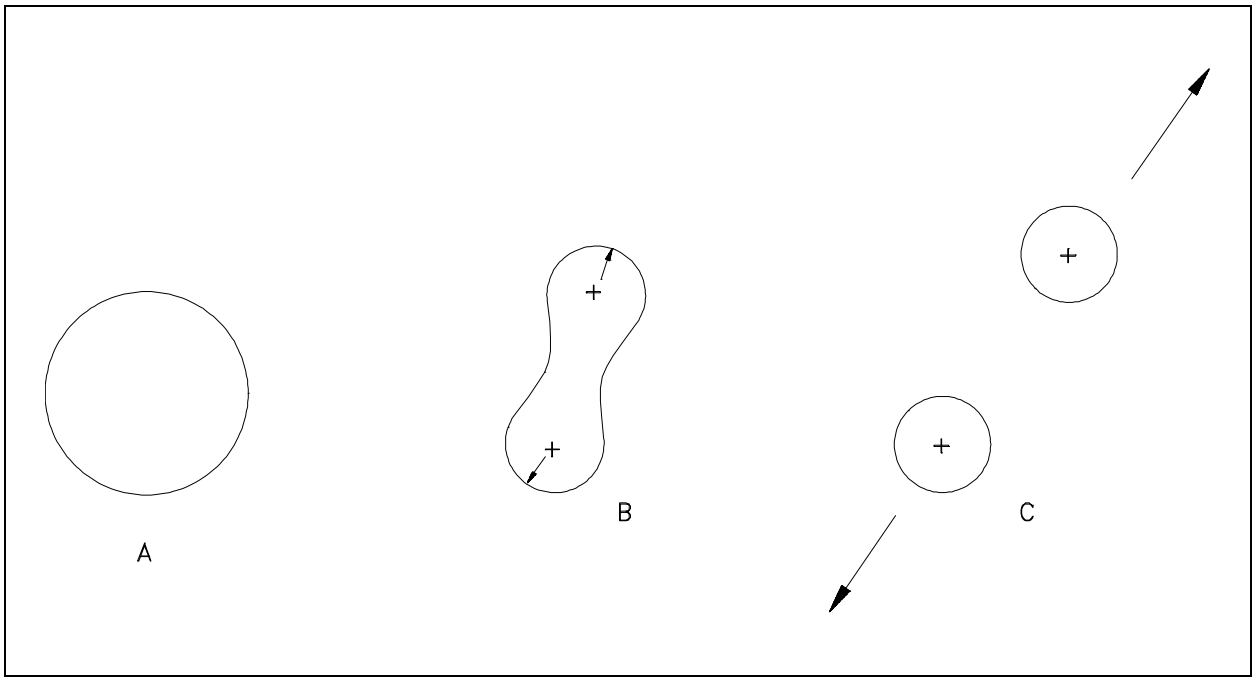


Figure 18 Liquid Drop Model of Fission

Critical Energy

The measure of how far the energy level of a nucleus is above its ground state is called the *excitation energy* (E_{exc}). For fission to occur, the excitation energy must be above a particular value for that nuclide. The *critical energy* (E_{crit}) is the minimum excitation energy required for fission to occur.

Fissile Material

A *fissile material* is composed of nuclides for which fission is possible with neutrons of any energy level. What is especially significant about these nuclides is their ability to be fissioned with zero kinetic energy neutrons (thermal neutrons). Thermal neutrons have very low kinetic energy levels (essentially zero) because they are roughly in equilibrium with the thermal motion of surrounding materials. Therefore, in order to be classified as fissile, a material must be capable of fissioning after absorbing a thermal neutron. Consequently, they impart essentially no kinetic energy to the reaction. Fission is possible in these materials with thermal neutrons, since the change in binding energy supplied by the neutron addition alone is high enough to exceed the critical energy. Some examples of fissile nuclides are uranium-235, uranium-233, and plutonium-239.

Fissionable Material

A *fissionable material* is composed of nuclides for which fission with neutrons is possible. All fissile nuclides fall into this category. However, also included are those nuclides that can be fissioned only with high energy neutrons. The change in binding energy that occurs as the result of neutron absorption results in a nuclear excitation energy level that is less than the required critical energy. Therefore, the additional excitation energy must be supplied by the kinetic energy of the incident neutron. The reason for this difference between fissile and fissionable materials is the so-called odd-even effect for nuclei. It has been observed that nuclei with even numbers of neutrons and/or protons are more stable than those with odd numbers. Therefore, adding a neutron to change a nucleus with an odd number of neutrons to a nucleus with an even number of neutrons produces an appreciably higher binding energy than adding a neutron to a nucleus already possessing an even number of neutrons. Some examples of nuclides requiring high energy neutrons to cause fission are thorium-232, uranium-238, and plutonium-240. Table 4 indicates the critical energy (E_{crit}) and the binding energy change for an added neutron (BE_n) to target nuclei of interest. For fission to be possible, the change in binding energy plus the kinetic energy must equal or exceed the critical energy ($\Delta BE + KE \geq E_{crit}$).

TABLE 4
Critical Energies Compared to Binding Energy of Last Neutron

Target Nucleus	Critical Energy E_{crit}	Binding Energy of Last Neutron BE_n	$BE_n - E_{crit}$
$^{232}_{90}Th$	7.5 MeV	5.4 MeV	-2.1 MeV
$^{238}_{92}U$	7.0 MeV	5.5 MeV	-1.5 MeV
$^{235}_{92}U$	6.5 MeV	6.8 MeV	+0.3 MeV
$^{233}_{92}U$	6.0 MeV	7.0 MeV	+1.0 MeV
$^{239}_{94}Pu$	5.0 MeV	6.6 MeV	+1.6 MeV

Uranium-235 fissions with thermal neutrons because the binding energy released by the absorption of a neutron is greater than the critical energy for fission; therefore uranium-235 is a fissile material. The binding energy released by uranium-238 absorbing a thermal neutron is less than the critical energy, so additional energy must be possessed by the neutron for fission to be possible. Consequently, uranium-238 is a fissionable material.

Fertile Material

All of the neutron absorption reactions that do not result in fission lead to the production of new nuclides through the process known as *transmutation*. These nuclides can, in turn, be transmuted again or may undergo radioactive decay to produce still different nuclides. The nuclides that are produced by this process are referred to as transmutation products. Because several of the fissile nuclides do not exist in nature, they can only be produced by nuclear reactions (transmutation). The target nuclei for such reactions are said to be fertile. *Fertile materials* are materials that can undergo transmutation to become fissile materials. Figure 19 traces the transmutation mechanism by which two fertile nuclides, thorium-232 and uranium-238, produce uranium-233 and plutonium-239, respectively.

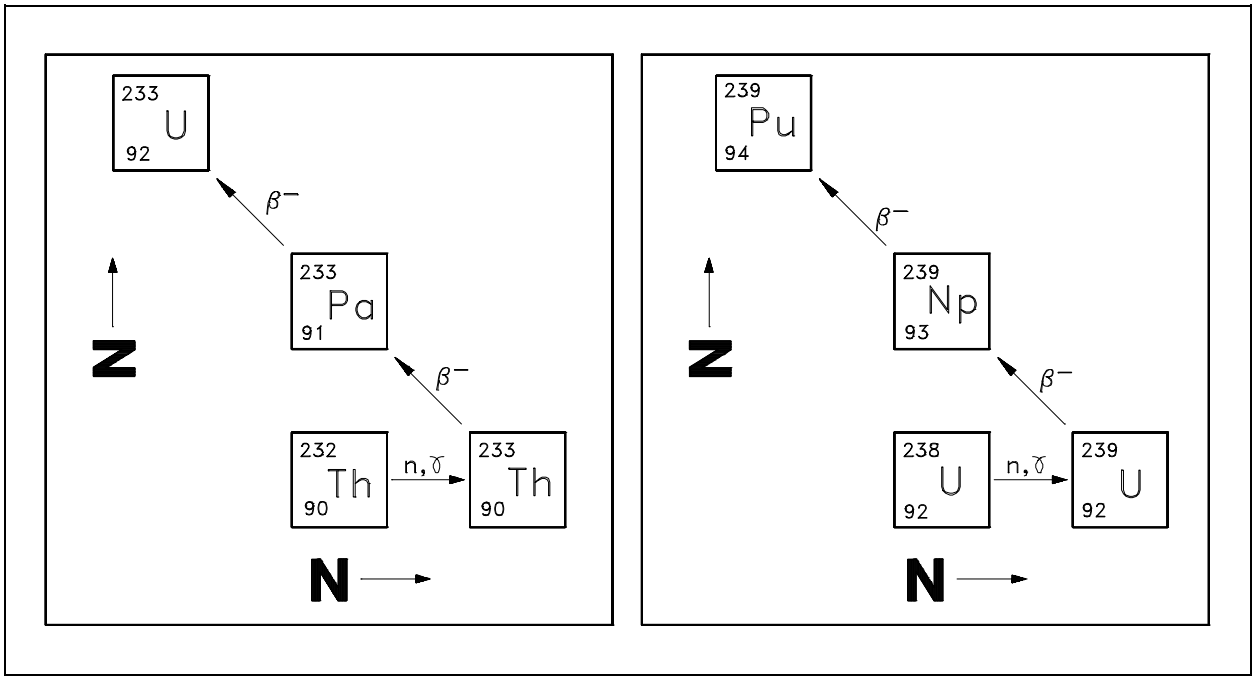


Figure 19 Conversion of Fertile Nuclides to Fissile Nuclides

If a reactor contains fertile material in addition to its fissile fuel, some new fuel will be produced as the original fuel is burned up. This is called *conversion*. Reactors that are specifically designed to produce fissionable fuel are called "breeder" reactors. In such reactors, the amount of fissionable fuel produced is greater than the amount of fuel burnup. If less fuel is produced than used, the process is called conversion, and the reactor is termed a "converter."

Binding Energy Per Nucleon (BE/A)

As the number of particles in a nucleus increases, the total binding energy also increases. The rate of increase, however, is not uniform. This lack of uniformity results in a variation in the amount of binding energy associated with each nucleon within the nucleus. This variation in the binding energy per nucleon (BE/A) is easily seen when the average BE/A is plotted versus atomic mass number (A), as shown in Figure 20.

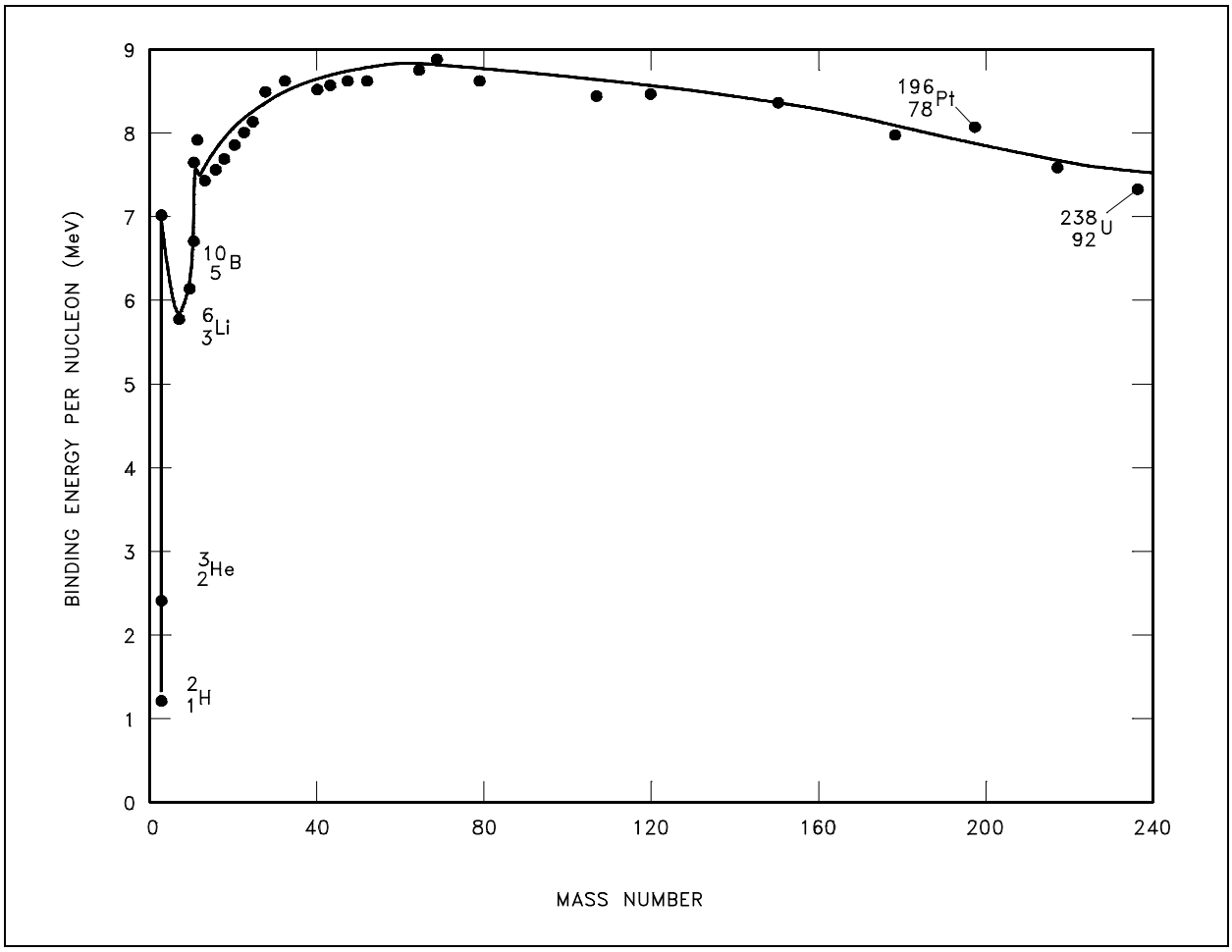


Figure 20 Binding Energy per Nucleon vs. Mass Number

Figure 20 illustrates that as the atomic mass number increases, the binding energy per nucleon decreases for A > 60. The BE/A curve reaches a maximum value of 8.79 MeV at A = 56 and decreases to about 7.6 MeV for A = 238. The general shape of the BE/A curve can be explained using the general properties of nuclear forces. The nucleus is held together by very short-range attractive forces that exist between nucleons. On the other hand, the nucleus is being forced apart by long range repulsive electrostatic (coulomb) forces that exist between all the protons in the nucleus.

As the atomic number and the atomic mass number increase, the repulsive electrostatic forces within the nucleus increase due to the greater number of protons in the heavy elements. To overcome this increased repulsion, the proportion of neutrons in the nucleus must increase to maintain stability. This increase in the neutron-to-proton ratio only partially compensates for the growing proton-proton repulsive force in the heavier, naturally occurring elements. Because the repulsive forces are increasing, less energy must be supplied, on the average, to remove a nucleon from the nucleus. The BE/A has decreased. The BE/A of a nucleus is an indication of its degree of stability. Generally, the more stable nuclides have higher BE/A than the less stable ones. The increase in the BE/A as the atomic mass number decreases from 260 to 60 is the primary reason for the energy liberation in the fission process. In addition, the increase in the BE/A as the atomic mass number increases from 1 to 60 is the reason for the energy liberation in the fusion process, which is the opposite reaction of fission.

The heaviest nuclei require only a small distortion from a spherical shape (small energy addition) for the relatively large coulomb forces forcing the two halves of the nucleus apart to overcome the attractive nuclear forces holding the two halves together. Consequently, the heaviest nuclei are easily fissionable compared to lighter nuclei.

Summary

The important information in this chapter is summarized on the following page.

Nuclear Fission Summary

- The fission process can be explained using the liquid drop model of a nucleus. In the ground state the nucleus is nearly spherical in shape. After the absorption of a neutron, the nucleus will be in an excited state and start to oscillate and become distorted. If the oscillations cause the nucleus to become shaped like a dumbbell, the repulsive electrostatic forces will overcome the short-range attractive nuclear forces, and the nucleus will split in two.

- Excitation energy is the amount of energy a nucleus has above its ground state.

- Critical energy is the minimum excitation energy that a nucleus must have before it can fission.

- Fissile material is material for which fission is possible with neutrons that have zero kinetic energy. Fissionable material is material for which fission caused by neutron absorption is possible provided the kinetic energy added with the binding energy is greater than the critical energy. Fertile material is material that can undergo transmutation to become fissile material.

- Transmutation is the process of neutron absorption and subsequent decay, which changes one nuclide to another nuclide. Conversion is the process of transmuting fertile material into fissile material in a reactor, where the amount of fissile material produced is less than the amount of fissile material consumed. Breeding is the same as conversion, except the amount of fissile material produced is more than the amount of fissile material consumed.

- The curve of binding energy per nucleon increases quickly through the light nuclides and reaches a maximum at a mass number of about 56. The curve decreases slowly for mass numbers greater than 60.

- The heaviest nuclei are easily fissionable because they require only a small distortion from the spherical shape to allow the coulomb forces to overcoming the attractive nuclear force, forcing the two halves of the nucleus apart.

- Uranium-235 fissions with thermal neutrons because the binding energy released by the absorption of a neutron is greater than the critical energy for fission. The binding energy released by uranium-238 absorbing a neutron is less than the critical energy, so additional kinetic energy must be possessed by the neutron for fission to be possible.

ENERGY RELEASE FROM FISSION

Fission of heavy nuclides converts a small amount of mass into an enormous amount of energy. The amount of energy released by fission can be determined based on either the change in mass that occurs during the reaction or by the difference in binding energy per nucleon between the fissile nuclide and the fission products.

EO 4.8 CHARACTERIZE the fission products in terms of mass groupings and radioactivity.

EO 4.9 Given the nuclides involved and their masses, CALCULATE the energy released from fission.

EO 4.10 Given the curve of Binding Energy per nucleon versus mass number, CALCULATE the energy released from fission.

Calculation of Fission Energy

Nuclear fission results in the release of enormous quantities of energy. It is necessary to be able to calculate the amount of energy that will be produced. The logical manner in which to pursue this is to first investigate a typical fission reaction such as the one listed below.

$$ {}^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right) $$

It can be seen that when the compound nucleus splits, it breaks into two fission fragments, rubidium-93, cesium-140, and some neutrons. Both fission products then decay by multiple β^- emissions as a result of the high neutron-to-proton ratio possessed by these nuclides.

In most cases, the resultant fission fragments have masses that vary widely. Figure 21 gives the percent yield for atomic mass numbers. The most probable pair of fission fragments for the thermal fission of the fuel uranium-235 have masses of about 95 and 140. Note that the vertical axis of the fission yield curve is on a logarithmic scale. Therefore, the formation of fission fragments of mass numbers of about 95 and 140 is highly likely.

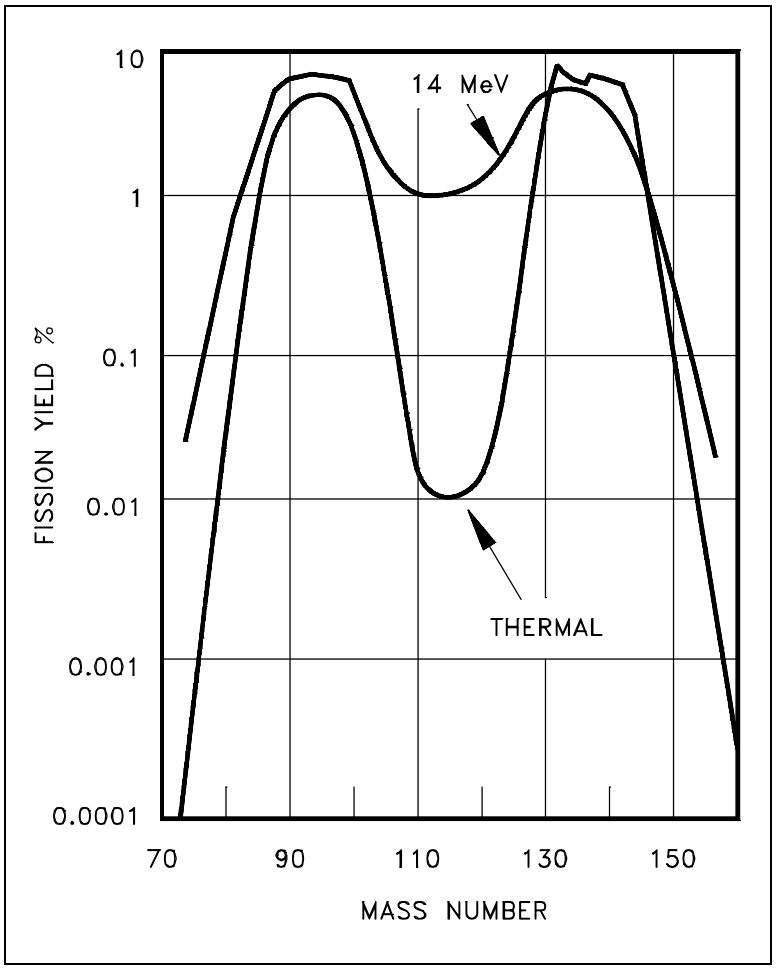


Figure 21 Uranium-235 Fission Yield vs. Mass Number

Referring now to the binding energy per nucleon curve (Figure 20), we can estimate the amount of energy released by our "typical" fission by plotting this reaction on the curve and calculating the change in binding energy (ΔBE) between the reactants on the left-hand side of the fission equation and the products on the right-hand side. Plotting the reactant and product nuclides on the curve shows that the total binding energy of the system after fission is greater than the total binding energy of the system before fission. When there is an increase in the total binding energy of a system, the system has become more stable by releasing an amount of energy equal to the increase in total binding energy of the system. Therefore, in the fission process, the energy liberated is equal to the increase in the total binding energy of the system.

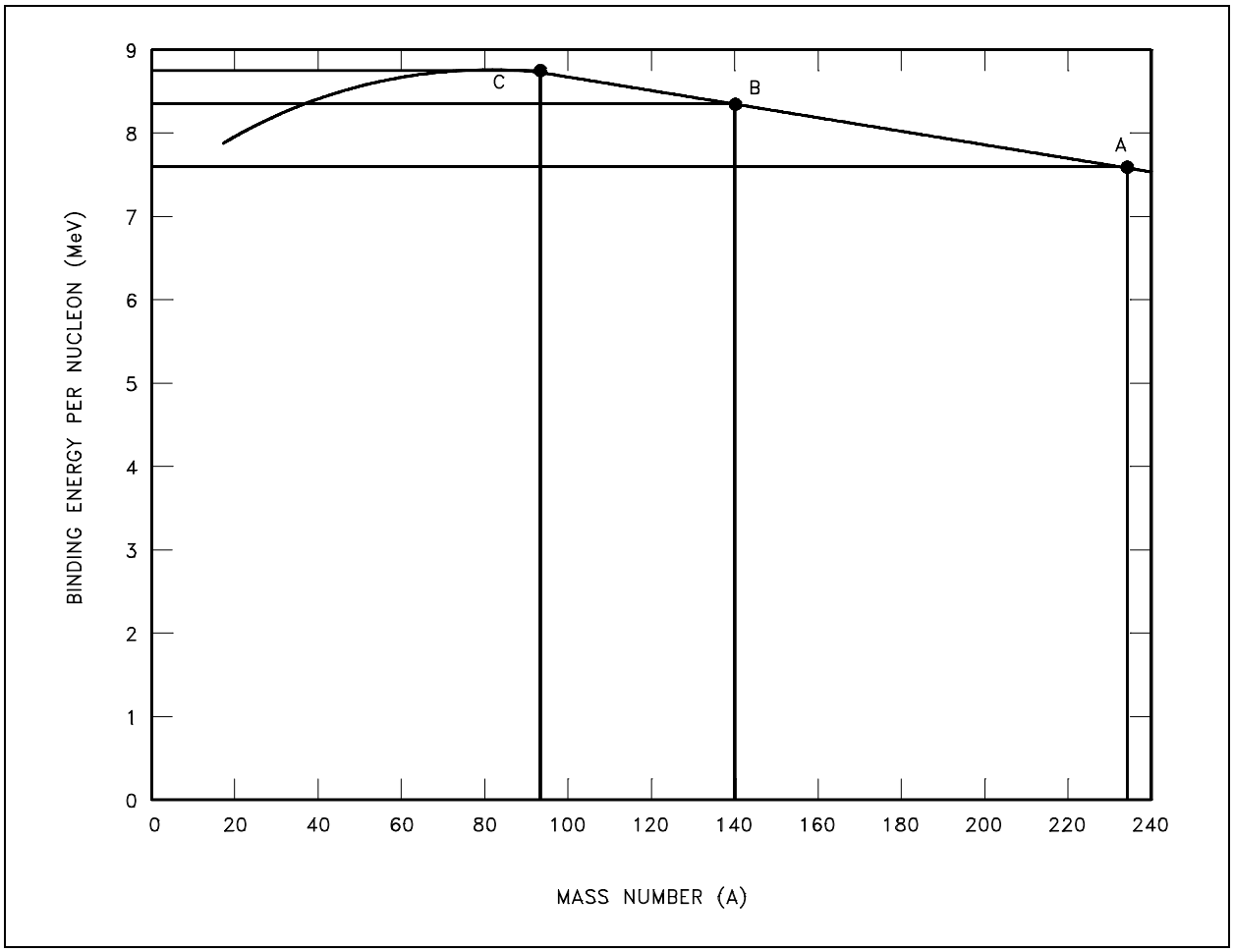


Figure 22 Change in Binding Energy for Typical Fission

Figure 22 graphically depicts that the binding energy per nucleon for the products (C, rubidium-93 and B, cesium-140) is greater than that for the reactant (A, uranium-235). The total binding energy for a nucleus can be found by multiplying the binding energy per nucleon by the number of nucleons.

TABLE 5
Binding Energies Calculated from Binding Energy per Nucleon Curve

Nuclide	B.E. per Nucleon (BE/A)	Mass Number (A)	Binding Energy (BE/A) x (A)
$^{93}_{37}$Rb	8.7 MeV	93	809 MeV
$^{140}_{55}$Cs	8.4 MeV	140	1176 MeV
$^{235}_{92}$U	7.6 MeV	235	1786 MeV

The energy released will be equivalent to the difference in binding energy (ΔBE) between the reactants and the products.

$$\begin{aligned}\Delta BE &= BE_{products} - BE_{reactants} \\ &= \left(BE_{Rb-93} + BE_{Cs-140}\right) - \left(BE_{U-235}\right) \\ &= (809\ MeV + 1176\ MeV) - 1786\ MeV \\ &= 199\ MeV\end{aligned}$$

The energy liberation during the fission process can also be explained from the standpoint of the conservation of mass-energy. During the fission process, there is a decrease in the mass of the system. There must, therefore, be energy liberated equal to the energy equivalent of the mass lost in the process. This method is more accurate than the previously illustrated method and is used when actually calculating the energy liberated during the fission process.

Again, referring to the "typical" fission reaction.

$$^{1}_{0}n + {}^{235}_{92}U \rightarrow \left({}^{236}_{92}U\right)^{*} \rightarrow {}^{140}_{55}Cs + {}^{93}_{37}Rb + 3\left({}^{1}_{0}n\right)$$

E_{Inst}, the instantaneous energy, is the energy released immediately after the fission process. It is equal to the energy equivalent of the mass lost in the fission process. It can be calculated as shown below.

Mass of the Reactants		Mass of the Products	
$^{235}_{92}U$	235.043924 amu	$^{93}_{37}Rb$	92.91699 amu
$^{1}_{0}n$	1.008665 amu	$^{140}_{55}Cs$	139.90910 amu
		$3\ (^{1}_{0}n)$	3.02599 amu
	236.052589 amu		235.85208 amu

$$\begin{aligned}\text{Mass difference} &= \text{Mass of Reactants} - \text{Mass of Products} \\ &= 236.052589\ amu - 235.85208\ amu \\ &= 0.200509\ amu\end{aligned}$$

This mass difference can be converted to an energy equivalent.

$$\begin{aligned}E_{Inst} &= 0.020059\ amu\left(\frac{931.5\ MeV}{amu}\right) \\ &= 186.8\ MeV\end{aligned}$$

The total energy released per fission will vary from the fission to the next depending on what fission products are formed, but the average total energy released per fission of uranium-235 with a thermal neutron is 200 MeV.

As illustrated in the preceding example, the majority of the energy liberated in the fission process is released immediately after the fission occurs and appears as the kinetic energy of the fission fragments, kinetic energy of the fission neutrons, and instantaneous gamma rays. The remaining energy is released over a period of time after the fission occurs and appears as kinetic energy of the beta, neutrino, and decay gamma rays.

Estimation of Decay Energy

In addition to this instantaneous energy release during the actual fission reaction, there is additional energy released when the fission fragments decay by β^- emission. This additional energy is called decay energy, E_{Decay}. The decay chains for rubidium-93 and cesium-140 are shown below.

$$^{93}_{37}\mathrm{Rb} \xrightarrow{\beta^-} {}^{93}_{38}\mathrm{Sr} \xrightarrow{\beta^-} {}^{93}_{39}\mathrm{Y} \xrightarrow{\beta^-} {}^{93}_{40}\mathrm{Zr} \xrightarrow{\beta^-} {}^{93}_{41}\mathrm{Nb}$$

$$^{140}_{55}\mathrm{Cs} \xrightarrow{\beta^-} {}^{140}_{56}\mathrm{Ba} \xrightarrow{\beta^-} {}^{140}_{57}\mathrm{La} \xrightarrow{\beta^-} {}^{140}_{58}\mathrm{Ce}$$

The energy released during the decay for each chain will be equivalent to the mass difference between the original fission product and the sum of the final stable nuclide and the beta particles emitted.

The energy released in the decay chain of rubidium-93 is calculated below.

$$E_{Decay} = \left[m_{Rb-93} - \left(m_{Nb-93} + 4\ m_{electron}\right)\right]\left(\frac{931.5\mathrm{MeV}}{\mathrm{amu}}\right)$$

$$= \left[92.91699\ \mathrm{amu} - \left(92.90638\ \mathrm{amu} + 4\ \left(0.0005486\ \mathrm{amu}\right)\right)\right]\left(\frac{931.5\ \mathrm{MeV}}{\mathrm{amu}}\right)$$

$$= 0.008416\ \mathrm{amu}\left(\frac{931.5\ \mathrm{MeV}}{\mathrm{amu}}\right)$$

$$= 7.84\ \mathrm{MeV}$$

The energy released in the decay chain of cesium-140 is calculated below.

$$E_{Decay} = \left[m_{Rb-93} - \left(m_{Nb-93} + 3\ m_{electron}\right)\right]\left(\frac{931.5MeV}{amu}\right)$$

$$= \left[139.90910\ amu - \left(139.90543\ amu + 3\ \left(0.0005486\ amu\right)\right)\right]\left(\frac{931.5\ MeV}{amu}\right)$$

$$= 0.000202\ amu\left(\frac{931.5\ MeV}{amu}\right)$$

$$= 1.89\ MeV$$

The total decay energy is the sum of the energies of the two chains, or 9.73 MeV.

Distribution of Fission Energy

The average energy distribution for the energy released per fission with a thermal neutron in uranium-235 is shown in Tables 6 and 7.

TABLE 6
Instantaneous Energy from Fission

Kinetic Energy of Fission Products	167 Mev
Energy of Fission Neutrons	5 MeV
Instantaneous Gamma-ray Energy	5 MeV
Capture Gamma-ray Energy	10 MeV
Total Instantaneous Energy	187 MeV

TABLE 7
Delayed Energy from Fission

Beta Particles From Fission Products	7 Mev
Gamma-rays from Fission Products	6 MeV
Neutrinos	10 MeV
Total Delayed Energy	23 MeV

Because the 10 MeV of neutrino energy shown in Table 7 is not absorbed in the reactor, the average value of 200 MeV per fission is still accurate. Note in Table 6 that some fission neutrons undergo radiative capture and the resultant gamma ray emission provides an additional 10 MeV of instantaneous energy, which contributes to the total of 187 MeV instantaneous energy.

All of the energy released, with the exception of the neutrino energy, is ultimately transformed into heat through a number of processes. The fission fragments, with their high positive charge and kinetic energy, cause ionization directly as they rip orbital electrons from the surrounding atoms. In this ionization process, kinetic energy is transferred to the surrounding atoms of the fuel material, resulting in an increase in temperature. The beta particles and gamma rays also give up their energy through ionization, and the fission neutrons interact and lose their energy through elastic scattering. Of the 200 MeV released per fission, about seven percent (13 MeV) is released at some time after the instant of fission. When a reactor is shut down, fissions essentially cease, but energy is still being released from the decay of fission products. The heat produced by this decay energy is referred to as "decay heat." Although decay energy represents about seven percent of reactor heat production during reactor operation, once the reactor is shut down the decay heat production drops off quickly to a small fraction of its value while operating. The decay heat produced is significant, however, and systems must be provided to keep the reactor cool even after shutdown.

Summary

The important information in this chapter is summarized below.

Energy Release From Fission Summary

- Fission products have some general characteristics in common.
  They generally decay by β^- emission.
  The most common mass numbers are grouped near 95 and 140.
- The energy released by fission can be calculated based on the difference in mass between the masses of the reactants before fission and the fission fragments and fission neutrons after fission.
- Another method to determine the energy released by fission is based on the change in binding energy per nucleon between the fissile nuclide and the fission products.

INTERACTION OF RADIATION WITH MATTER

Different types of radiation interact with matter in widely different ways. A large, massive, charged alpha particle cannot penetrate a piece of paper and even has a limited range in dry air. A neutrino, at the other extreme, has a low probability of interacting with any matter, even if it passed through the diameter of the earth.

EO 5.1 DESCRIBE interactions of the following with matter:

a. Alpha particle
b. Beta particle
c. Positron
d. Neutron

EO 5.2 DESCRIBE the following ways that gamma radiation interacts with matter:

a. Photoelectric effect
b. Compton scattering
c. Pair production

Interaction of Radiation With Matter

Radiation can be classified into two general groups, charged and uncharged; therefore, it may be expected that interactions with matter fall into two general types. Charged particles directly ionize the media through which they pass, while uncharged particles and photons can cause ionization only indirectly or by secondary radiation.

A moving charged particle has an electrical field surrounding it, which interacts with the atomic structure of the medium through which it is passing. This interaction decelerates the particle and accelerates electrons in the atoms of the medium. The accelerated electrons may acquire enough energy to escape from the parent atom. This process, whereby radiation "strips" off orbital electrons, is called ionization. Uncharged moving particles have no electrical field, so they can only lose energy and cause ionization by such means as collisions or scattering. A photon can lose energy by the photoelectric effect, Compton effect, or pair production.

Because ionizing radiation creates ions in pairs, the intensity of ionization or the specific ionization is defined as the number of ion-pairs formed per centimeter of travel in a given material. The amount of ionization produced by a charged particle per unit path length, which is a measure of its ionizing power, is roughly proportional to the particle's mass and the square of its charge as illustrated in the equation below.

$$I = \frac{m\ z^2}{K.E.}$$

where:

I is the ionizing power
m is the mass of the particle
z is the number of unit charges it carries
K.E. is its kinetic energy

Since m for an alpha particle is about 7300 times as large as m for a beta particle, and z is twice as great, an alpha will produce much more ionization per unit path length than a beta particle of the same energy. This phenomenon occurs because the larger alpha particle moves slower for a given energy and thus acts on a given electron for a longer time.

Alpha Radiation

Alpha radiation is normally produced from the radioactive decay of heavy nuclides and from certain nuclear reactions. The alpha particle consists of 2 neutrons and 2 protons, so it is essentially the same as the nucleus of a helium atom. Because it has no electrons, the alpha particle has a charge of +2. This positive charge causes the alpha particle to strip electrons from the orbits of atoms in its vicinity. As the alpha particle passes through material, it removes electrons from the orbits of atoms it passes near. Energy is required to remove electrons and the energy of the alpha particle is reduced by each reaction. Eventually the particle will expend its kinetic energy, gain 2 electrons in orbit, and become a helium atom. Because of its strong positive charge and large mass, the alpha particle deposits a large amount of energy in a short distance of travel. This rapid, large deposition of energy limits the penetration of alpha particles. The most energetic alpha particles are stopped by a few centimeters of air or a sheet of paper.

Beta-Minus Radiation

A beta-minus particle is an electron that has been ejected at a high velocity from an unstable nucleus. An electron has a small mass and an electrical charge of -1. Beta particles cause ionization by displacing electrons from atom orbits. The ionization occurs from collisions with orbiting electrons. Each collision removes kinetic energy from the beta particle, causing it to slow down. Eventually the beta particle will be slowed enough to allow it to be captured as an

orbiting electron in an atom. Although more penetrating than the alpha, the beta is relatively easy to stop and has a low power of penetration. Even the most energetic beta radiation can be stopped by a few millimeters of metal.

Positron Radiation

Positively charged electrons are called positrons. Except for the positive charge, they are identical to beta-minus particles and interact with matter in a similar manner. Positrons are very short-lived, however, and quickly are annihilated by interaction with a negatively charged electron, producing two gammas with a combined energy (calculated below) equal to the rest mass of the positive and negative electrons.

$$2 \text{ electrons} \left(\frac{0.000549 \text{ amu}}{\text{electron}} \right) \left(\frac{931.5 \text{ MeV}}{\text{amu}} \right) = 1.02 \text{ MeV}$$

Neutron Radiation

Neutrons have no electrical charge. They have nearly the same mass as a proton (a hydrogen atom nucleus). A neutron has hundreds of times more mass than an electron, but 1/4 the mass of an alpha particle. The source of neutrons is primarily nuclear reactions, such as fission, but they may also be produced from the decay of radioactive nuclides. Because of its lack of charge, the neutron is difficult to stop and has a high penetrating power.

Neutrons are attenuated (reduced in energy and numbers) by three major interactions, elastic scatter, inelastic scatter, and absorption. In elastic scatter, a neutron collides with a nucleus and bounces off. This reaction transmits some of the kinetic energy of the neutron to the nucleus of the atom, resulting in the neutron being slowed, and the atom receives some kinetic energy (motion). This process is sometimes referred to as "the billiard ball effect."

As the mass of the nucleus approaches the mass of the neutron, this reaction becomes more effective in slowing the neutron. Hydrogenous material attenuates neutrons most effectively.

In the inelastic scatter reaction, the same neutron/nucleus collision occurs as in elastic scatter. However, in this reaction, the nucleus receives some internal energy as well as kinetic energy. This slows the neutron, but leaves the nucleus in an excited state. When the nucleus decays to its original energy level, it normally emits a gamma ray.

In the absorption reaction, the neutron is actually absorbed into the nucleus of an atom. The neutron is captured, but the atom is left in an excited state. If the nucleus emits one or more gamma rays to reach a stable level, the process is called radiative capture. This reaction occurs at most neutron energy levels, but is more probable at lower energy levels.

Gamma Radiation

Gamma radiation is electromagnetic radiation. It is commonly referred to as a gamma ray and is very similar to an x-ray. The difference is that gamma rays are emitted from the nucleus of an atom, and x-rays are produced by orbiting electrons. The x-ray is produced when orbiting electrons move to a lower energy orbit or when fast-moving electrons approaching an atom are deflected and decelerated as they react with the atom's electrical field (called Bremsstrahlung). The gamma ray is produced by the decay of excited nuclei and by nuclear reactions. Because the gamma ray has no mass and no charge, it is difficult to stop and has a very high penetrating power. A small fraction of the original gamma stream will pass through several feet of concrete or several meters of water.

There are three methods of attenuating gamma rays. The first method is referred to as the photo-electric effect. When a low energy gamma strikes an atom, the total energy of the gamma is expended in ejecting an electron from orbit. The result is ionization of the atom and expulsion of a high energy electron. This reaction is most predominant with low energy gammas interacting in materials with high atomic weight and rarely occurs with gammas having an energy above 1 MeV. Annihilation of the gamma results. Any gamma energy in excess of the binding energy of the electron is carried off by the electron in the form of kinetic energy.

The second method of attenuation of gammas is called Compton scattering. The gamma interacts with an orbital or free electron; however, in this case, the photon loses only a fraction of its energy. The actual energy loss depending on the scattering angle of the gamma. The gamma continues on at lower energy, and the energy difference is absorbed by the electron. This reaction becomes important for gamma energies of about 0.1 MeV and higher.

At higher energy levels, a third method of attenuation is predominant. This method is pair-production. When a high energy gamma passes close enough to a heavy nucleus, the gamma completely disappears, and an electron and a positron are formed. For this reaction to take place, the original gamma must have at least 1.02 MeV energy. Any energy greater than 1.02 MeV becomes kinetic energy shared between the electron and positron. The probability of pair-production increases significantly for higher energy gammas.

Summary

The important information in this chapter is summarized below.

Interaction of Radiation with Matter Summary

- An alpha particle deposits a large amount of energy in a short distance of travel due to its large mass and charge.
- Beta-minus particles interact with the electrons orbiting the nucleus of atoms, causing ionization by displacing the electrons. The beta particle loses energy with each interaction. After the beta particle loses enough energy, it is captured in the orbital shells of an atom.
- Positrons interact with matter much the same way as beta minus particles. After the positron has lost most of its energy by ionizing atoms, it is annihilated by interaction with an electron. The electron-positron pair disappear and are replaced by two gammas, each with the energy equivalent of the mass of an electron (0.51 MeV).
- Neutrons interact with matter by elastic scattering, inelastic scattering, or absorption.
- Photoelectric effect is where a gamma interacts with an electron orbiting an atom. The entire energy of the gamma is transferred to the electron, and the electron is ejected from its orbit.
- In Compton scattering a gamma interacts with an orbital electron, but only part of the gamma energy is transferred to the electron. The electron is ejected from its orbit, and the gamma is scattered off at a lower energy.
- In pair-production, a gamma interacts with the electric field of a nucleus and is converted into an electron-positron pair. The gamma must have an energy greater than 1.02 MeV for this to occur.

Intentionally Left Blank

Department of Energy
Fundamentals Handbook

NUCLEAR PHYSICS AND REACTOR THEORY
Module 2
Reactor Theory (Neutron Characteristics)

TABLE OF CONTENTS

LIST OF FIGURES iii

LIST OF TABLES iv

REFERENCES v

OBJECTIVES vi

NEUTRON SOURCES 1

- Neutron Sources 1
- Intrinsic Neutron Sources 1
- Installed Neutron Sources 3
- Summary 4

NUCLEAR CROSS SECTIONS AND NEUTRON FLUX 5

- Introduction 6
- Atom Density 6
- Cross Sections 7
- Mean Free Path 10
- Calculation of Macroscopic Cross Section and Mean Free Path 11
- Effects of Temperature on Cross Section 14
- Neutron Flux 15
- Self-Shielding 16
- Summary 16

REACTION RATES 18

- Reaction Rates 18
- Reactor Power Calculation 20
- Relationship Between Neutron Flux and Reactor Power 21
- Summary 22

NEUTRON MODERATION 23

- Neutron Slowing Down and Thermalization 23
- Macroscopic Slowing Down Power 26
- Moderating Ratio 27
- Summary 28

TABLE OF CONTENTS (Cont.)

PROMPT AND DELAYED NEUTRONS 29

Neutron Classification 29
Neutron Generation Time 30
Summary 31

NEUTRON FLUX SPECTRUM 33

Prompt Neutron Energies 33
Thermal and Fast Breeder Reactor Neutron Spectra 34
Most Probable Neutron Velocities 35
Summary 37

LIST OF FIGURES

Figure 1 Typical Neutron Absorption Cross Section vs. Neutron Energy 9

Figure 2 Prompt Fission Neutron Energy Spectrum for Thermal Fission of Uranium-235 33

Figure 3 Comparison of Neutron Flux Spectra for Thermal and Fast Breeder Reactor . . . 34

LIST OF TABLES

Table 1 Neutron Production by Spontaneous Fission 2

Table 2 Moderating Properties of Materials 27

Table 3 Delayed Neutron Precursor Groups for Thermal Fission in Uranium-235 30

REFERENCES

- Foster, Arthur R. and Wright, Robert L. Jr., Basic Nuclear Engineering, 3rd Edition, Allyn and Bacon, Inc., 1977.
- Jacobs, A.M., Kline, D.E., and Remick, F. J., Basic Principles of Nuclear Science and Reactors, Van Nostrand Company, Inc., 1960.
- Kaplan, Irving, Nuclear Physics, 2nd Edition, Addison-Wesley Company, 1962.
- Knief, Ronald Allen, Nuclear Energy Technology: Theory and Practice of Commercial Nuclear Power, McGraw-Hill, 1981.
- Lamarsh, John R., Introduction to Nuclear Engineering, Addison-Wesley Company, 1977.
- Lamarsh, John R., Introduction to Nuclear Reactor Theory, Addison-Wesley Company, 1972.
- General Electric Company, Nuclides and Isotopes: Chart of the Nuclides, 14th Edition, General Electric Company, 1989.
- Academic Program for Nuclear Power Plant Personnel, Volume III, Columbia, MD, General Physics Corporation, Library of Congress Card #A 326517, 1982.
- Glasstone, Samuel, Sourcebook on Atomic Energy, Robert F. Krieger Publishing Company, Inc., 1979.
- Glasstone, Samuel and Sesonske, Alexander, Nuclear Reactor Engineering, 3rd Edition, Van Nostrand Reinhold Company, 1981.

TERMINAL OBJECTIVE

1.0 Without references, **EXPLAIN** how neutron sources produce neutrons.

ENABLING OBJECTIVES

1.1 **DEFINE** the following terms:

a. Intrinsic neutron source
b. Installed neutron source

1.2 **LIST** three examples of reactions that produce neutrons in intrinsic neutron sources.

1.3 **LIST** three examples of reactions that produce neutrons in installed neutron sources.

TERMINAL OBJECTIVE

2.0 Given the necessary information for calculations, **EXPLAIN** basic concepts in reactor physics and perform calculations.

ENABLING OBJECTIVES

2.1 **DEFINE** the following terms:

a. Atom density
b. Neutron flux
c. Microscopic cross section
d. Barn
e. Macroscopic cross section
f. Mean free path

2.2 **EXPRESS** macroscopic cross section in terms of microscopic cross section.

2.3 **DESCRIBE** how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

2.4 **DESCRIBE** the cause of resonance absorption in terms of nuclear energy levels.

2.5 **DESCRIBE** the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

2.6 **EXPRESS** mean free path in terms of macroscopic cross section.

2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, **CALCULATE** the macroscopic cross section for a mixture.

2.8 **CALCULATE** a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

2.9 **EXPLAIN** neutron shadowing or self-shielding.

2.10 Given the neutron flux and macroscopic cross section, **CALCULATE** the reaction rate.

2.11 **DESCRIBE** the relationship between neutron flux and reactor power.

ENABLING OBJECTIVES (Cont.)

2.12 **DEFINE** the following concepts:

a. Thermalization
b. Moderator
c. Moderating ratio
d. Average logarithmic energy decrement
e. Macroscopic slowing down power

2.13 **LIST** three desirable characteristics of a moderator.

2.14 Given an average fractional energy loss per collision, **CALCULATE** the energy loss after a specified number of collisions.

TERMINAL OBJECTIVE

3.0 Without references, **EXPLAIN** the production process and effects on fission of prompt and delayed neutrons.

ENABLING OBJECTIVES

3.1 **STATE** the origin of prompt neutrons and delayed neutrons.

3.2 **STATE** the approximate fraction of neutrons that are born as delayed neutrons from the fission of the following nuclear fuels:

a. Uranium-235
b. Plutonium-239

3.3 **EXPLAIN** the mechanism for production of delayed neutrons.

3.4 **EXPLAIN** prompt and delayed neutron generation times.

3.5 Given prompt and delayed neutron generation times and delayed neutron fraction, **CALCULATE** the average generation time.

3.6 **EXPLAIN** the effect of delayed neutrons on reactor control.

TERMINAL OBJECTIVE

4.0 Without references, **DESCRIBE** the neutron energy spectrum for the type of reactor presented in this module.

ENABLING OBJECTIVES

4.1 **STATE** the average energy at which prompt neutrons are produced.

4.2 **DESCRIBE** the neutron energy spectrum in the following reactors:

a. Fast reactor
b. Thermal reactor

4.3 **EXPLAIN** the reason for the particular shape of the fast, intermediate, and slow energy regions of the neutron flux spectrum for a thermal reactor.

NEUTRON SOURCES

Neutrons from a variety of sources are always present in a reactor core. This is true even when the reactor is shut down. Some of these neutrons are produced by naturally occurring (intrinsic) neutron sources, while others may be the result of fabricated (installed) neutron sources that are incorporated into the design of the reactor. The neutrons produced by sources other than neutron-induced fission are often grouped together and classified as source neutrons.

EO 1.1 DEFINE the following terms:

a. Intrinsic neutron source
b. Installed neutron source

EO 1.2 LIST three examples of reactions that produce neutrons in intrinsic neutron sources.

EO 1.3 LIST three examples of reactions that produce neutrons in installed neutron sources.

Neutron Sources

In addition to neutron-induced fission, neutrons are produced by other reactions. The neutrons produced by reactions other than neutron-induced fission are called *source* neutrons. Source neutrons are important because they ensure that the neutron population remains high enough to allow a visible indication of neutron level on the most sensitive monitoring instruments while the reactor is shutdown and during the startup sequence. This verifies instrument operability and allows monitoring of neutron population changes. Source neutrons can be classified as either intrinsic or installed neutron sources.

Intrinsic Neutron Sources

Some neutrons will be produced in the materials present in the reactor due to a variety of unavoidable reactions that occur because of the nature of these materials. *Intrinsic neutron sources* are those neutron-producing reactions that always occur in reactor materials.

A limited number of neutrons will always be present, even in a reactor core that has never been operated, due to spontaneous fission of some heavy nuclides that are present in the fuel. Uranium-238, uranium-235, and plutonium-239 undergo spontaneous fission to a limited extent. Uranium-238, for example, yields almost 60 neutrons per hour per gram. Table 1 illustrates a comparison of the rate at which different heavy nuclides produce neutrons by spontaneous fission. Californium-252 is not an intrinsic neutron source, but will be discussed in the section on installed neutron sources.

TABLE 1
Neutron Production by Spontaneous Fission

Nuclide	$T_{1/2}$ (Fission)	$T_{1/2}$ (α-decay)	neutrons/sec/gram
$^{235}_{92}U$	1.8×10^{17} years	6.8×10^{8} years	8.0×10^{-4}
$^{238}_{92}U$	8.0×10^{15} years	4.5×10^{9} years	1.6×10^{-2}
$^{239}_{94}Pu$	5.5×10^{5} years	2.4×10^{4} years	3.0×10^{-2}
$^{240}_{94}Pu$	1.2×10^{11} years	6.6×10^{3} years	1.0×10^{3}
$^{252}_{98}Cf$	66.0 years	2.65 years	2.3×10^{12}

Another intrinsic neutron source is a reaction involving natural boron and fuel. In some reactors, natural boron is loaded into the reactor core as a neutron absorber to improve reactor control or increase core life-time. Boron-11 (80.1% of natural boron) undergoes a reaction with the alpha particle emitted by the radioactive decay of heavy nuclides in the fuel to yield a neutron as shown below.

$$^{11}_{5}B + ^{4}_{2}\alpha \rightarrow ^{14}_{7}N + ^{1}_{0}n$$

The boron-11 must be mixed with, or in very close proximity to, the fuel for this reaction because of the short path length of the alpha particle. For a reactor core with this configuration, this (α,n) reaction is an important source of neutrons for reactor startup.

In a reactor that has been operated, another source of neutrons becomes significant. Neutrons may be produced by the interaction of a gamma ray and a deuterium nucleus. This reaction is commonly referred to as a photoneutron reaction because it is initiated by electromagnetic radiation and results in the production of a neutron. The photoneutron reaction is shown below.

$$^{2}_{1}H + ^{0}_{0}\gamma \rightarrow \left(^{2}_{1}H\right)^{*} \rightarrow ^{1}_{1}H + ^{1}_{0}n$$

There is an abundant supply of high energy gammas in a reactor that has been operated because many of the fission products are gamma emitters. All water-cooled reactors have some deuterium present in the coolant in the reactor core because a small fraction of natural hydrogen is the isotope deuterium. The atom percentage of deuterium in the water ranges from close to the naturally occurring value (0.015%) for light water reactors to above 90% deuterium for heavy water reactors. Therefore, the required conditions for production of photoneutrons exist. The supply of gamma rays decreases with time after shutdown as the gamma emitters decay; therefore, the photoneutron production rate also decreases. In a few particular reactors, additional D_2O (heavy water) may be added to the reactor to increase the production of photoneutrons following a long shutdown period.

Installed Neutron Sources

Because intrinsic neutron sources can be relatively weak or dependent upon the recent power history of the reactor, many reactors have artificial sources of neutrons installed. These neutron sources ensure that shutdown neutron levels are high enough to be detected by the nuclear instruments at all times. This provides a true picture of reactor conditions and any change in these conditions. An *installed neutron source* is an assembly placed in or near the reactor for the sole purpose of producing source neutrons.

One strong source of neutrons is the artificial nuclide californium-252, which emits neutrons at the rate of about 2×10^{12} neutrons per second per gram as the result of spontaneous fission. Important drawbacks for some applications may be its high cost and its short half-life (2.65 years).

Many installed neutron sources use the (α,n) reaction with beryllium. These sources are composed of a mixture of metallic beryllium (100% beryllium-9) with a small quantity of an alpha particle emitter, such as a compound of radium, polonium, or plutonium. The reaction that occurs is shown below.

$$ {}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\alpha \rightarrow \left({}^{13}_{6}\mathrm{C}\right)^{*} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n} $$

The beryllium is intimately (homogeneously) mixed with the alpha emitter and is usually enclosed in a stainless steel capsule.

Another type of installed neutron source that is widely used is a photoneutron source that employs the (γ,n) reaction with beryllium. Beryllium is used for photoneutron sources because its stable isotope beryllium-9 has a weakly attached last neutron with a binding energy of only 1.66 MeV. Thus, a gamma ray with greater energy than 1.66 MeV can cause neutrons to be ejected by the (γ,n) reaction as shown below.

$$ {}^{9}_{4}\mathrm{Be} + {}^{0}_{0}\gamma \rightarrow \left({}^{9}_{4}\mathrm{Be}\right)^{*} \rightarrow {}^{8}_{4}\mathrm{Be} + {}^{1}_{0}\mathrm{n} $$

Many startup sources of this type use antimony and beryllium because after activation with neutrons the radioactive antimony becomes an emitter of high energy gammas. The photoneutron sources of this type are constructed somewhat differently from the (α,n) types. One design incorporates a capsule of irradiated antimony enclosed in a beryllium sleeve. The entire assembly is then encased in a stainless steel cladding. A large reactor may have several neutron sources of this type installed within the core.

Summary

The important information in this chapter is summarized below.

Neutron Sources Summary

- Intrinsic neutron sources are sources of neutrons from materials that are in the reactor for other purposes such as fuel, burnable poison, or moderator.
- Installed neutron sources are materials or components placed in the reactor specifically for the purpose of producing source neutrons.
- Examples of intrinsic neutron sources are listed below.

  Spontaneous fission of heavy nuclides in fuel, such as uranium-238, uranium-235, and plutonium-239, results in fission fragments and free neutrons.

  Boron-11 mixed with the fuel undergoes an alpha-neutron reaction and becomes nitrogen-14.

  Deuterium present in the reactor coolant undergoes a gamma-neutron reaction and becomes hydrogen-1.
- Examples of installed neutron sources are listed below.

  Spontaneous fission of californium-252 results in fission fragments and free neutrons.

  Beryllium-9 undergoes an alpha-neutron reaction (alpha from the decay of plutonium, polonium, or radium) and becomes carbon-12.

  Beryllium-9 undergoes a gamma-neutron reaction (high energy gamma from decay of antimony-124) and becomes beryllium-8.

NUCLEAR CROSS SECTIONS AND NEUTRON FLUX

To determine the frequency of neutron interactions, it is necessary to describe the availability of neutrons to cause interaction and the probability of a neutron interacting with material. The availability of neutrons and the probability of interaction are quantified by the neutron flux and nuclear cross section.

EO 2.1 DEFINE the following terms:

a. Atom density
b. Neutron flux
c. Microscopic cross section
d. Barn
e. Macroscopic cross section
f. Mean free path

EO 2.2 EXPRESS macroscopic cross section in terms of microscopic cross section.

EO 2.3 DESCRIBE how the absorption cross section of typical nuclides varies with neutron energy at energies below the resonance absorption region.

EO 2.4 DESCRIBE the cause of resonance absorption in terms of nuclear energy levels.

EO 2.5 DESCRIBE the energy dependence of resonance absorption peaks for typical light and heavy nuclei.

EO 2.6 EXPRESS mean free path in terms of macroscopic cross section.

EO 2.7 Given the number densities (or total density and component fractions) and microscopic cross sections of components, CALCULATE the macroscopic cross section for a mixture.

EO 2.8 CALCULATE a macroscopic cross section given a material density, atomic mass, and microscopic cross section.

EO 2.9 EXPLAIN neutron shadowing or self-shielding.

Introduction

Fission neutrons are born with an average energy of about 2 MeV. These fast neutrons interact with the reactor core materials in various absorption and scattering reactions. Collisions that result in scattering are useful in slowing neutrons to thermal energies. Thermal neutrons may be absorbed by fissile nuclei to produce more fissions or be absorbed in fertile material for conversion to fissionable fuel. Absorption of neutrons in structural components, coolant, and other non-fuel material results in the removal of neutrons without fulfilling any useful purpose.

To safely and efficiently operate a nuclear reactor it is necessary to predict the probability that a particular absorption or scattering reaction will occur. Once these probabilities are known, if the availability of neutrons can be determined, then the rate at which these nuclear reactions take place can be predicted.

Atom Density

One important property of a material is the atom density. The *atom density* is the number of atoms of a given type per unit volume of the material. To calculate the atom density of a substance use Equation (2-1).

$$N = \frac{\rho N_A}{M} \qquad (2\text{-}1)$$

where:

N = atom density (atoms/cm^3)
ρ = density (g/cm^3)
N_A = Avogadro's number (6.022 x 10^{23} atoms/mole)
M = gram atomic weight

Example:

A block of aluminum has a density of 2.699 g/cm^3. If the gram atomic weight of aluminum is 26.9815 g, calculate the atom density of the aluminum.

Solution:

$$N = \frac{\rho N_A}{M}$$

$$= \frac{2.699 \ \frac{g}{cm^3} \left(6.022 \times 10^{23} \ \frac{atoms}{mole} \right)}{26.9815 \ \frac{g}{mole}}$$

$$= 6.024 \times 10^{22} \ \frac{atoms}{cm^3}$$

Cross Sections

The probability of a neutron interacting with a nucleus for a particular reaction is dependent upon not only the kind of nucleus involved, but also the energy of the neutron. Accordingly, the absorption of a thermal neutron in most materials is much more probable than the absorption of a fast neutron. Also, the probability of interaction will vary depending upon the type of reaction involved.

The probability of a particular reaction occurring between a neutron and a nucleus is called the *microscopic cross section* (σ) of the nucleus for the particular reaction. This cross section will vary with the energy of the neutron. The microscopic cross section may also be regarded as the effective area the nucleus presents to the neutron for the particular reaction. The larger the effective area, the greater the probability for reaction.

Because the microscopic cross section is an area, it is expressed in units of area, or square centimeters. A square centimeter is tremendously large in comparison to the effective area of a nucleus, and it has been suggested that a physicist once referred to the measure of a square centimeter as being "as big as a barn" when applied to nuclear processes. The name has persisted and microscopic cross sections are expressed in terms of *barns*. The relationship between barns and cm^2 is shown below.

1 barn = 10^{-24} cm^2

Whether a neutron will interact with a certain volume of material depends not only on the microscopic cross section of the individual nuclei but also on the number of nuclei within that volume. Therefore, it is necessary to define another kind of cross section known as the macroscopic cross section (Σ). The *macroscopic cross section* is the probability of a given reaction occurring per unit travel of the neutron. Σ is related to the microscopic cross section (σ) by the relationship shown below.

$$\Sigma = N \sigma \qquad (2\text{-}2)$$

where:

Σ = macroscopic cross section (cm^{-1})
N = atom density of material ($atoms/cm^3$)
σ = microscopic cross-section (cm^2)

The difference between the microscopic and macroscopic cross sections is extremely important and is restated for clarity. The microscopic cross section (σ) represents the effective target area that a single nucleus presents to a bombarding particle. The units are given in barns or cm^2. The macroscopic cross section (Σ) represents the effective target area that is presented by all of the nuclei contained in 1 cm^3 of the material. The units are given as 1/cm or cm^{-1}.

A neutron interacts with an atom of the material it enters in two basic ways. It will either interact through a scattering interaction or through an absorption reaction. The probability of a neutron being absorbed by a particular atom is the microscopic cross section for absorption, σ_a. The probability of a neutron scattering off of a particular nucleus is the microscopic cross section for scattering, σ_s. The sum of the microscopic cross section for absorption and the microscopic cross section for scattering is the total microscopic cross section, σ_T.

$$\sigma_T = \sigma_a + \sigma_s$$

Both the absorption and the scattering microscopic cross sections can be further divided. For instance, the scattering cross section is the sum of the elastic scattering cross section (σ_{se}) and the inelastic scattering cross section (σ_{si}).

$$\sigma_s = \sigma_{se} + \sigma_{si}$$

The microscopic absorption cross section (σ_a) includes all reactions except scattering. However, for most purposes it is sufficient to merely separate it into two categories, fission (σ_f) and capture (σ_c). Radiative capture of neutrons was described in the Neutron Interactions chapter of Module 1.

$$\sigma_a = \sigma_f + \sigma_c$$

The variation of absorption cross sections with neutron energy is often complicated. For many elements the absorption cross sections are small, ranging from a fraction of a barn to a few barns for slow (or thermal) neutrons.

For a considerable number of nuclides of moderately high (or high) mass numbers, an examination of the variation of the absorption cross section with the energy of the incident neutron reveals the existence of three regions on a curve of absorption cross section versus neutron energy. This cross section is illustrated in Figure 1. First, the cross section decreases steadily with increasing neutron energy in a low energy region, which includes the thermal range (E < 1 eV). In this region the absorption cross section, which is often high, is inversely proportional to the velocity (v). This region is frequently referred to as the "1/v region," because the absorption cross section is proportional to 1/v, which is the reciprocal of neutron velocity. Following the 1/v region, there occurs the "resonance region" in which the cross sections rise sharply to high values called "resonance peaks" for neutrons of certain energies, and then fall again. These energies are called resonance energies and are a result of the affinity of the nucleus for neutrons whose energies closely match its discrete, quantum energy levels. That is, when the binding energy of a neutron plus the kinetic energy of the neutron are exactly equal to the amount required to raise a compound nucleus from its ground state to a quantum level, resonance absorption occurs. The following example problem further illustrates this point.

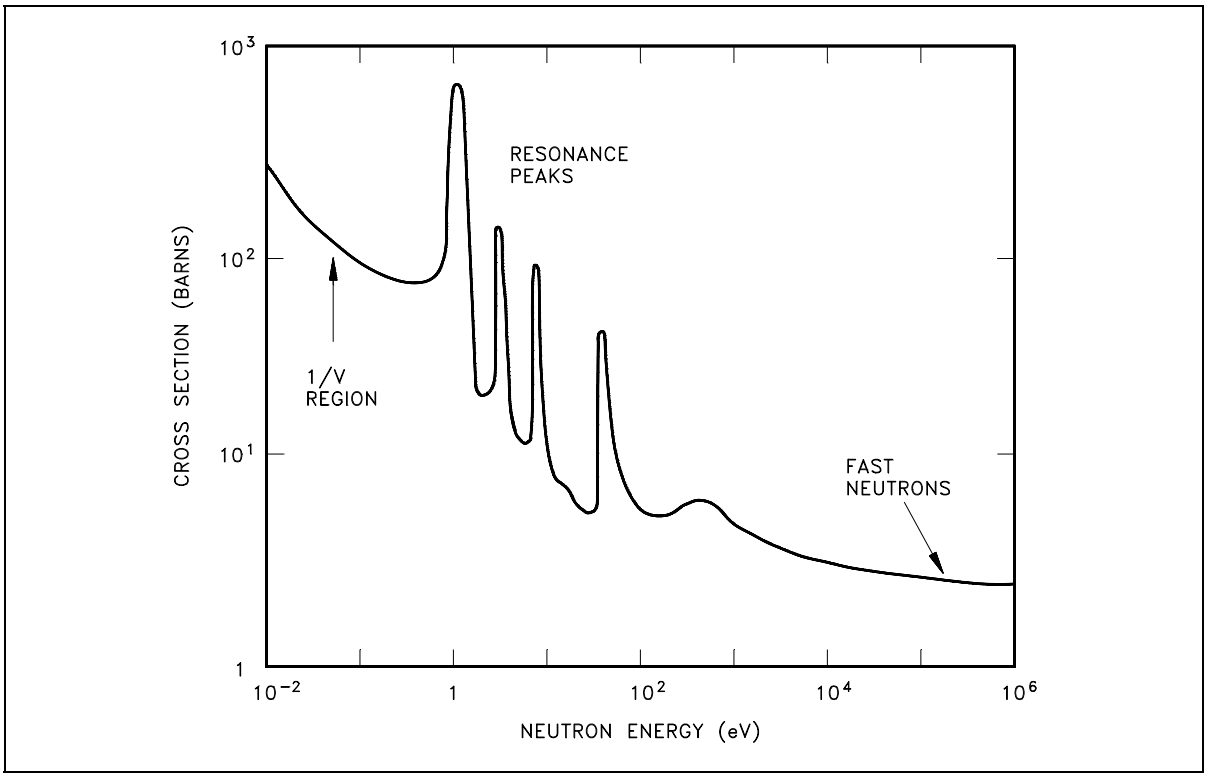


Figure 1 Typical Neutron Absorption Cross Section vs. Neutron Energy

Assuming that uranium-236 has a nuclear quantum energy level at 6.8 MeV above its ground state, calculate the kinetic energy a neutron must possess to undergo resonant absorption in uranium-235 at this resonance energy level.

BE = [Mass(^{235}U) + Mass(neutron) - Mass(^{236}U)] x 931 MeV/amu

BE = (235.043925 + 1.008665 - 236.045563) x 931 MeV/amu

BE = (0.007025 amu) x 931 MeV/amu = 6.54 MeV

6.8 MeV - 6.54 MeV = 0.26 MeV

The difference between the binding energy and the quantum energy level equals the amount of kinetic energy the neutron must possess. The typical heavy nucleus will have many closely-spaced resonances starting in the low energy (eV) range. This is because heavy nuclei are complex and have more possible configurations and corresponding energy states. Light nuclei, being less complex, have fewer possible energy states and fewer resonances that are sparsely distributed at higher energy levels.

For higher neutron energies, the absorption cross section steadily decreases as the energy of the neutron increases. This is called the "fast neutron region." In this region the absorption cross sections are usually less than 10 barns.

With the exception of hydrogen, for which the value is fairly large, the elastic scattering cross sections are generally small, for example, 5 barns to 10 barns. This is close to the magnitude of the actual geometric cross sectional area expected for atomic nuclei. In potential scattering, the cross section is essentially constant and independent of neutron energy. Resonance elastic scattering and inelastic scattering exhibit resonance peaks similar to those associated with absorption cross sections. The resonances occur at lower energies for heavy nuclei than for light nuclei. In general, the variations in scattering cross sections are very small when compared to the variations that occur in absorption cross sections.

Mean Free Path

If a neutron has a certain probability of undergoing a particular interaction in one centimeter of travel, then the inverse of this value describes how far the neutron will travel (in the average case) before undergoing an interaction. This average distance traveled by a neutron before interaction is known as the *mean free path* for that interaction and is represented by the symbol λ. The relationship between the mean free path (λ) and the macroscopic cross section (Σ) is shown below.

$$\lambda = \frac{1}{\Sigma} \qquad (2\text{-}3)$$

Calculation of Macroscopic Cross Section and Mean Free Path

Most materials are composed of several elements, and because most elements are composed of several isotopes, most materials involve many cross sections, one for each isotope involved. Therefore, to include all the isotopes within a given material, it is necessary to determine the macroscopic cross section for each isotope and then sum all the individual macroscopic cross sections. Equation (2-4) can be used to determine the macroscopic cross section for a composite material.

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots N_n \sigma_n \qquad (2\text{-}4)$$

where:

N_n = the number of nuclei per cm^3 of the n^{th} element
σ_n = the microscopic cross section of the n^{th} element

The following example problems illustrate the calculation of the macroscopic cross section for a single element and for combinations of materials.

Example 1:

Find the macroscopic thermal neutron absorption cross section for iron, which has a density of 7.86 g/cm^3. The microscopic cross section for absorption of iron is 2.56 barns and the gram atomic weight is 55.847 g.

Solution:

Step 1: Using Equation (2-1), calculate the atom density of iron.

$$N = \frac{\rho N_A}{M}$$

$$= \frac{7.86 \ \frac{g}{cm^3} \left(6.022 \times 10^{23} \ \frac{atoms}{mole} \right)}{55.847 \ \frac{g}{mole}}$$

$$= 8.48 \times 10^{22} \ \frac{atoms}{cm^3}$$

Step 2: Use this atom density in Equation (2-2) to calculate the macroscopic cross section.

$$\Sigma_a = N \sigma_a$$

$$= 8.48 \times 10^{22} \ \frac{atoms}{cm^3} \ (2.56 \text{ barns}) \left(\frac{1 \times 10^{-24} \ cm^2}{1 \text{ barn}} \right)$$

$$= 0.217 \ cm^{-1}$$

Example 2:

An alloy is composed of 95% aluminum and 5% silicon (by weight). The density of the alloy is 2.66 g/cm^3. Properties of aluminum and silicon are shown below.

Element	Gram Atomic Weight	$\sigma_{a\ (barns)}$	$\sigma_{s\ (barns)}$
Aluminum	26.9815	0.23	1.49
Silicon	28.0855	0.16	2.20

1. Calculate the atom densities for the aluminum and silicon.

2. Determine the absorption and scattering macroscopic cross sections for thermal neutrons.

3. Calculate the mean free paths for absorption and scattering.

Solution:

Step 1: The density of the aluminum will be 95% of the total density. Using Equation (2-1) yields the atom densities.

$$N_{Al} = \frac{\rho_{Al}\, N_A}{M_{Al}}$$

$$= \frac{0.95 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022 \times 10^{23}\ \frac{atoms}{mole}\right)}{26.9815\ \frac{g}{mole}}$$

$$= 5.64 \times 10^{22}\ \frac{atoms}{cm^3}$$

$$N_{Si} = \frac{\rho_{Si}\, N_A}{M_{Si}}$$

$$= \frac{0.05 \left(2.66\ \frac{g}{cm^3}\right) \left(6.022 \times 10^{23}\ \frac{atoms}{mole}\right)}{28.0855\ \frac{g}{mole}}$$

$$= 2.85 \times 10^{21}\ \frac{atoms}{cm^3}$$

Step 2: The macroscopic cross sections for absorption and scattering are calculated using Equation (2-4).

$$\Sigma_a = N_{Al}\,\sigma_{a,Al} + N_{Si}\,\sigma_{a,Si}$$

$$= \left(5.64 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.23 \times 10^{-24}\ \text{cm}^2\right) + \left(2.85 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(0.16 \times 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0134\ \text{cm}^{-1}$$

$$\Sigma_s = N_{Al}\,\sigma_{s,Al} + N_{Si}\,\sigma_{s,Si}$$

$$= \left(5.64 \times 10^{22}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(1.49 \times 10^{-24}\ \text{cm}^2\right) + \left(2.85 \times 10^{21}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(2.20 \times 10^{-24}\ \text{cm}^2\right)$$

$$= 0.0903\ \text{cm}^{-1}$$

Step 3: The mean free paths are calculated by inserting the macroscopic cross sections calculated above into Equation (2-3).

$$\lambda_a = \frac{1}{\Sigma_a}$$

$$= \frac{1}{0.01345\ \text{cm}^{-1}}$$

$$= 74.3\ \text{cm}$$

$$\lambda_s = \frac{1}{\Sigma_s}$$

$$= \frac{1}{0.0903\ \text{cm}^{-1}}$$

$$= 11.1\ \text{cm}$$

Thus, a neutron must travel an average of 74.3 cm to interact by absorption in this alloy, but it must travel only 11.1 cm to interact by scattering.

Effects of Temperature on Cross Section

As discussed, the microscopic absorption cross section varies significantly as neutron energy varies. The microscopic cross sections provided on most charts and tables are measured for a standard neutron velocity of 2200 meters/second, which corresponds to an ambient temperature of 68°F. Therefore, if our material is at a higher temperature, the absorption cross section will be lower than the value for 68°F, and any cross sections which involve absorption (for example, σ_a, σ_c, σ_f) must be corrected for the existing temperature.

The following formula is used to correct microscopic cross sections for temperature. Although the example illustrates absorption cross section, the same formula may be used to correct capture and fission cross sections.

$$\sigma = \sigma_o \left(\frac{T_o}{T} \right)^{1/2}$$

where:

σ = microscopic cross section corrected for temperature
σ_o = microscopic cross section at reference temperature (68°F or 20°C)
T_o = reference temperature (68°F) in degrees Rankine (°R) or Kelvin (°K)
T = temperature for which corrected value is being calculated

NOTE: When using this formula, all temperatures must be converted to °R or °K.

°R = °F + 460
°K = °C + 273

Example:

What is the value of σ_f for uranium-235 for thermal neutrons at 500°F? Uranium-235 has a σ_f of 583 barns at 68°F.

Solution:

$$\sigma_f = \sigma_{f,o} \left(\frac{T_o}{T} \right)^{1/2}$$

$$= 583 \text{ barns} \left(\frac{68°F + 460}{500°F + 460} \right)^{1/2}$$

$$= 432 \text{ barns}$$

Neutron Flux

Macroscopic cross sections for neutron reactions with materials determine the probability of one neutron undergoing a specific reaction per centimeter of travel through that material. If one wants to determine how many reactions will actually occur, it is necessary to know how many neutrons are traveling through the material and how many centimeters they travel each second.

It is convenient to consider the number of neutrons existing in one cubic centimeter at any one instant and the total distance they travel each second while in that cubic centimeter. The number of neutrons existing in a cm^3 of material at any instant is called *neutron density* and is represented by the symbol n with units of neutrons/cm^3. The total distance these neutrons can travel each second will be determined by their velocity.

A good way of defining *neutron flux* (ϕ) is to consider it to be the total path length covered by all neutrons in one cubic centimeter during one second. Mathematically, this is the equation below.

$$\phi = n v \qquad (2\text{-}5)$$

where:

ϕ = neutron flux (neutrons/cm^2-sec)
n = neutron density (neutrons/cm^3)
v = neutron velocity (cm/sec)

The term neutron flux in some applications (for example, cross section measurement) is used as parallel beams of neutrons traveling in a single direction. The *intensity* (I) of a neutron beam is the product of the neutron density times the average neutron velocity. The directional beam intensity is equal to the number of neutrons per unit area and time (neutrons/cm^2-sec) falling on a surface perpendicular to the direction of the beam.

One can think of the neutron flux in a reactor as being comprised of many neutron beams traveling in various directions. Then, the neutron flux becomes the scalar sum of these directional flux intensities (added as numbers and not vectors), that is, $\phi = I_1 + I_2 + I_3 + ...I_n$. Since the atoms in a reactor do not interact preferentially with neutrons from any particular direction, all of these directional beams contribute to the total rate of reaction. In reality, at a given point within a reactor, neutrons will be traveling in all directions.

Self-Shielding

In some locations within the reactor, the flux level may be significantly lower than in other areas due to a phenomenon referred to as *neutron shadowing* or *self-shielding*. For example, the interior of a fuel pin or pellet will "see" a lower average flux level than the outer surfaces since an appreciable fraction of the neutrons will have been absorbed and therefore cannot reach the interior of the fuel pin. This is especially important at resonance energies, where the absorption cross sections are large.

Summary

The important information in this chapter is summarized below.

Nuclear Cross Section and Neutron Flux Summary

- Atom density (N) is the number of atoms of a given type per unit volume of material.
- Microscopic cross section (σ) is the probability of a given reaction occurring between a neutron and a nucleus.
- Microscopic cross sections are measured in units of barns, where 1 barn = 10^{-24} cm^2.
- Macroscopic cross section (Σ) is the probability of a given reaction occurring per unit length of travel of the neutron. The units for macroscopic cross section are cm^{-1}.
- The mean free path (λ) is the average distance that a neutron travels in a material between interactions.
- Neutron flux (ϕ) is the total path length traveled by all neutrons in one cubic centimeter of material during one second.
- The macroscopic cross section for a material can be calculated using the equation below.

  $$\Sigma = N \sigma$$

- The absorption cross section for a material usually has three distinct regions. At low neutron energies (<1 eV) the cross section is inversely proportional to the neutron velocity.
- Resonance absorption occurs when the sum of the kinetic energy of the neutron and its binding energy is equal to an allowed nuclear energy level of the nucleus.
- Resonance peaks exist at intermediate energy levels. For higher neutron energies, the absorption cross section steadily decreases as the neutron energy increases.
- The mean free path equals $1/\Sigma$.
- The macroscopic cross section for a mixture of materials can be calculated using the equation below.

  $$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + N_3 \sigma_3 + \ldots\ldots N_n \sigma_n$$

- Self-shielding is where the local neutron flux is depressed within a material due to neutron absorption near the surface of the material.

REACTION RATES

It is possible to determine the rate at which a nuclear reaction will take place based on the neutron flux, cross section for the interaction, and atom density of the target. This relationship illustrates how a change in one of these items affects the reaction rate.

EO 2.10 **Given the neutron flux and macroscopic cross section, CALCULATE the reaction rate.**

EO 2.11 **DESCRIBE the relationship between neutron flux and reactor power.**

Reaction Rates

If the total path length of all the neutrons in a cubic centimeter in a second is known, (neutron flux (ϕ)), and if the probability of having an interaction per centimeter path length is also known (macroscopic cross section (Σ)), multiply them together to get the number of interactions taking place in that cubic centimeter in one second. This value is known as the reaction rate and is denoted by the symbol R. The reaction rate can be calculated by the equation shown below.

$$R = \phi \Sigma \quad (2\text{-}6)$$

where:

R = reaction rate (reactions/sec)
ϕ = neutron flux (neutrons/cm^2-sec)
Σ = macroscopic cross section (cm^{-1})

Substituting the fact that $\Sigma = N \sigma$ into Equation (2-6) yields the equation below.

$$\mathbf{R = \phi N \sigma}$$

where:

σ = microscopic cross section (cm^2)
N = atom density (atoms/cm^3)

The reaction rate calculated will depend on which macroscopic cross section is used in the calculation. Normally, the reaction rate of greatest interest is the fission reaction rate.

Example:

If a one cubic centimeter section of a reactor has a macroscopic fission cross section of 0.1 cm^{-1}, and if the thermal neutron flux is 10^{13} neutrons/cm^2-sec, what is the fission rate in that cubic centimeter?

Solution:

$$R_f = \phi \Sigma_f$$

$$= \left(1 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}\right)\left(0.1 \text{ cm}^{-1}\right)$$

$$= 1 \times 10^{12} \frac{\text{fissions}}{\text{cm}^3\text{-sec}}$$

In addition to using Equation (2-6) to determine the reaction rate based on the physical properties of material, it is also possible to algebraically manipulate the equation to determine physical properties if the reaction rate is known.

Example:

A reactor operating at a flux level of 3 x 10^{13} neutrons/cm^2-sec contains 10^{20} atoms of uranium-235 per cm^3. The reaction rate is 1.29 x 10^{12} fission/cm^3. Calculate Σ_f and σ_f.

Solution:

Step 1: The macroscopic cross section can be determined by solving Equation (2-6) for Σ_f and substituting the appropriate values.

$$R_f = \phi \Sigma_f$$

$$\Sigma_f = \frac{R_f}{\phi}$$

$$= \frac{1.29 \times 10^{12} \frac{\text{fissions}}{\text{cm}^3\text{-sec}}}{3 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}}$$

$$= 0.043 \text{ cm}^{-1}$$

Step 2: To find the microscopic cross section, replace Σ_f with (N x σ_f) and solve for σ_f.

$$R_f = \phi\ N\ \sigma_f$$

$$\sigma_f = \frac{R_f}{N\ \phi}$$

$$= \frac{1.29\ x\ 10^{12}\ \frac{\text{fissions}}{\text{cm}^3\text{-sec}}}{\left(1\ x\ 10^{20}\ \frac{\text{atoms}}{\text{cm}^3}\right)\left(3\ x\ 10^{13}\ \frac{\text{neutrons}}{\text{cm}^2\text{-sec}}\right)}$$

$$= 4.3\ x\ 10^{-22}\ \text{cm}^2\left(\frac{1\ \text{barn}}{1\ x\ 10^{-24}\ \text{cm}^2}\right)$$

$$= 430\ \text{barns}$$

Reactor Power Calculation

Multiplying the reaction rate per unit volume by the total volume of the core results in the total number of reactions occurring in the core per unit time. If the amount of energy involved in each reaction were known, it would be possible to determine the rate of energy release (power) due to a certain reaction.

In a reactor where the average energy per fission is 200 MeV, it is possible to determine the number of fissions per second that are necessary to produce one watt of power using the following conversion factors.

1 fission = 200 MeV
1 MeV = 1.602 x 10^{-6} ergs
1 erg = 1 x 10^{-7} watt-sec

$$1\ \text{watt}\left(\frac{1\ \text{erg}}{1\ x\ 10^{-7}\ \text{watt-sec}}\right)\left(\frac{1\ \text{MeV}}{1.602\ x\ 10^{-6}\text{erg}}\right)\left(\frac{1\ \text{fission}}{200\ \text{MeV}}\right) = 3.12\ x\ 10^{10}\ \frac{\text{fissions}}{\text{second}}$$

This is equivalent to stating that 3.12 x 10^{10} fissions release 1 watt-second of energy.

The power released in a reactor can be calculated based on Equation (2-6). Multiplying the reaction rate by the volume of the reactor results in the total fission rate for the entire reactor. Dividing by the number of fissions per watt-sec results in the power released by fission in the reactor in units of watts. This relationship is shown mathematically in Equation (2-7) below.

$$P = \frac{\phi_{th}\ \Sigma_f\ V}{3.12\ x\ 10^{10}\ \frac{\text{fissions}}{\text{watt-sec}}} \qquad (2\text{-}7)$$

where:

P = power (watts)
ϕ_{th} = thermal neutron flux (neutrons/cm^2-sec)
Σ_f = macroscopic cross section for fission (cm^{-1})
V = volume of core (cm^3)

Relationship Between Neutron Flux and Reactor Power

In an operating reactor the volume of the reactor is constant. Over a relatively short period of time (days or weeks), the number density of the fuel atoms is also relatively constant. Since the atom density and microscopic cross section are constant, the macroscopic cross section must also be constant. Examining Equation (2-7), it is apparent that if the reactor volume and macroscopic cross section are constant, then the reactor power and the neutron flux are directly proportional. This is true for day-to-day operation. The neutron flux for a given power level will increase very slowly over a period of months due to the burnup of the fuel and resulting decrease in atom density and macroscopic cross section.

Summary

The important information in this chapter is summarized below.

Reaction Rates Summary

- The reaction rate is the number of interactions of a particular type occurring in a cubic centimeter of material in a second.
- The reaction rate can be calculated by the equation below.

$$R = \phi \Sigma$$

- Over a period of several days, while the atom density of the fuel can be considered constant, the neutron flux is directly proportional to reactor power.

NEUTRON MODERATION

In thermal reactors, the neutrons that cause fission are at a much lower energy than the energy level at which they were born from fission. In this type of reactor, specific materials must be included in the reactor design to reduce the energy level of the neutrons in an efficient manner.

EO 2.12 DEFINE the following concepts:

a. Thermalization
b. Moderator
c. Moderating ratio
d. Average logarithmic energy decrement
e. Macroscopic slowing down power

EO 2.13 LIST three desirable characteristics of a moderator.

EO 2.14 Given an average fractional energy loss per collision, CALCULATE the energy loss after a specified number of collisions.

Neutron Slowing Down and Thermalization

Fission neutrons are produced at an average energy level of 2 MeV and immediately begin to slow down as the result of numerous scattering reactions with a variety of target nuclei. After a number of collisions with nuclei, the speed of a neutron is reduced to such an extent that it has approximately the same average kinetic energy as the atoms (or molecules) of the medium in which the neutron is undergoing elastic scattering. This energy, which is only a small fraction of an electron volt at ordinary temperatures (0.025 eV at 20°C), is frequently referred to as the thermal energy, since it depends upon the temperature. Neutrons whose energies have been reduced to values in this region (< 1 eV) are designated thermal neutrons. The process of reducing the energy of a neutron to the thermal region by elastic scattering is referred to as *thermalization*, slowing down, or moderation. The material used for the purpose of thermalizing neutrons is called a *moderator*. A good moderator reduces the speed of neutrons in a small number of collisions, but does not absorb them to any great extent. Slowing the neutrons in as few collisions as possible is desirable in order to reduce the amount of neutron leakage from the core and also to reduce the number of resonance absorptions in non-fuel materials. Neutron leakage and resonance absorption will be discussed in the next module.

The ideal moderating material (moderator) should have the following nuclear properties.

- large scattering cross section
- small absorption cross section
- large energy loss per collision

A convenient measure of energy loss per collision is the logarithmic energy decrement. The *average logarithmic energy decrement* is the average decrease per collision in the logarithm of the neutron energy. This quantity is represented by the symbol ξ (Greek letter xi).

$$\xi = \ln E_i - \ln E_f$$

$$\xi = \ln \left(\frac{E_i}{E_f} \right) \quad (2\text{-}8)$$

where:

ξ = average logarithmic energy decrement
E_i = average initial neutron energy
E_f = average final neutron energy

The symbol ξ is commonly called the average logarithmic energy decrement because of the fact that a neutron loses, on the average, a fixed fraction of its energy per scattering collision. Since the fraction of energy retained by a neutron in a single elastic collision is a constant for a given material, ξ is also a constant. Because it is a constant for each type of material and does not depend upon the initial neutron energy, ξ is a convenient quantity for assessing the moderating ability of a material.

The values for the lighter nuclei are tabulated in a variety of sources. The following commonly used approximation may be used when a tabulated value is not available.

$$\xi = \frac{2}{A + \frac{2}{3}}$$

This approximation is relatively accurate for mass numbers (A) greater than 10, but for some low values of A it may be in error by over three percent.

Since ξ represents the average logarithmic energy loss per collision, the total number of collisions necessary for a neutron to lose a given amount of energy may be determined by dividing ξ into the difference of the natural logarithms of the energy range in question. The number of collisions (N) to travel from any energy, E_{high}, to any lower energy, E_{low}, can be calculated as shown below.

$$N = \frac{\ln E_{high} - \ln E_{low}}{\xi}$$

$$= \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

Example:

How many collisions are required to slow a neutron from an energy of 2 MeV to a thermal energy of 0.025 eV, using water as the moderator? Water has a value of 0.948 for ξ.

Solution:

$$N = \frac{\ln \left(\frac{E_{high}}{E_{low}} \right)}{\xi}$$

$$= \frac{\ln \left(\frac{2 \times 10^6 \text{ eV}}{0.025 \text{ eV}} \right)}{0.948}$$

$$= 19.2 \text{ collisions}$$

Sometimes it is convenient, based upon information known, to work with an average fractional energy loss per collision as opposed to an average logarithmic fraction. If the initial neutron energy level and the average fractional energy loss per collision are known, the final energy level for a given number of collisions may be computed using the following formula.

$$E_N = E_o (1 - x)^N \quad (2\text{-}9)$$

where:

E_o = initial neutron energy
E_N = neutron energy after N collisions
x = average fractional energy loss per collision
N = number of collisions

Example:

If the average fractional energy loss per collision in hydrogen is 0.63, what will be the energy of a 2 MeV neutron after (a) 5 collisions? (b) 10 collisions?

Solution:

a)

$$E_N = E_o (1 - x)^N$$
$$E_5 = (2 \times 10^6 \text{ eV}) (1 - 0.63)^5$$
$$= 13.9 \text{ keV}$$

b)

$$E_N = E_o (1 - x)^N$$
$$E_{10} = (2 \times 10^6 \text{ eV}) (1 - 0.63)^{10}$$
$$= 96.2 \text{ eV}$$

Macroscopic Slowing Down Power

Although the logarithmic energy decrement is a convenient measure of the ability of a material to slow neutrons, it does not measure all necessary properties of a moderator. A better measure of the capabilities of a material is the macroscopic slowing down power. The *macroscopic slowing down power* (MSDP) is the product of the logarithmic energy decrement and the macroscopic cross section for scattering in the material. Equation (2-10) illustrates how to calculate the macroscopic slowing down power.

$$MSDP = \xi \Sigma_s \quad (2\text{-}10)$$

Moderating Ratio

Macroscopic slowing down power indicates how rapidly a neutron will slow down in the material in question, but it still does not fully explain the effectiveness of the material as a moderator. An element such as boron has a high logarithmic energy decrement and a good slowing down power, but it is a poor moderator because of its high probability of absorbing neutrons.

The most complete measure of the effectiveness of a moderator is the moderating ratio. The *moderating ratio* is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption. The higher the moderating ratio, the more effectively the material performs as a moderator. Equation (2-11) shows how to calculate the moderating ratio of a material.

$$MR = \frac{\xi \, \Sigma_s}{\Sigma_a} \qquad (2\text{-}11)$$

Moderating properties of different materials are compared in Table 2.

TABLE 2
Moderating Properties of Materials

Material	ξ	Number of Collisions to Thermalize	Macroscopic Slowing Down Power	Moderating Ratio
H_2O	0.927	19	1.425	62
D_2O	0.510	35	0.177	4830
Helium	0.427	42	9×10^{-6}	51
Beryllium	0.207	86	0.154	126
Boron	0.171	105	0.092	0.00086
Carbon	0.158	114	0.083	216

Summary

The important information in this chapter is summarized below.

Neutron Moderation Summary

- Thermalization is the process of reducing the energy level of a neutron from the energy level at which it is produced to an energy level in the thermal range.
- The moderator is the reactor material that is present for the purpose of thermalizing neutrons.
- Moderating ratio is the ratio of the macroscopic slowing down power to the macroscopic cross section for absorption.
- The average logarithmic energy decrement (ξ) is the average change in the logarithm of neutron energy per collision.
- Macroscopic slowing down power is the product of the average logarithmic energy decrement and the macroscopic cross section for scattering.
- There are three desirable characteristics of a moderator.

  1. large scattering cross section
  2. small absorption cross section
  3. large energy loss per collision

- The energy loss after a specified number of collisions can be calculated using the equation below.

$$E_N = E_o (1 - x)^N$$

PROMPT AND DELAYED NEUTRONS

Not all neutrons are released at the same time following fission. Most neutrons are released virtually instantaneously and are called prompt neutrons. A very small fraction of neutrons are released after the decay of fission products and are called delayed neutrons. Although delayed neutrons are a very small fraction of the total number of neutrons, they play an extremely important role in the control of the reactor.

EO 3.1 **STATE the origin of prompt neutrons and delayed neutrons.**

EO 3.2 **STATE the approximate fraction of neutrons that are born as delayed neutrons from the fission of the following nuclear fuels:**

a. Uranium-235
b. Plutonium-239

EO 3.3 **EXPLAIN the mechanism for production of delayed neutrons.**

EO 3.4 **EXPLAIN prompt and delayed neutron generation times.**

EO 3.5 **Given prompt and delayed neutron generation times and delayed neutron fraction, CALCULATE the average generation time.**

EO 3.6 **EXPLAIN the effect of delayed neutrons on reactor control.**

Neutron Classification

The great majority (over 99%) of the neutrons produced in fission are released within about 10^{-13} seconds of the actual fission event. These are called *prompt neutrons*. A small portion of fission neutrons are *delayed neutrons*, which are produced for some time after the fission process has taken place. The delayed neutrons are emitted immediately following the first beta decay of a fission fragment known as a delayed neutron precursor. An example of a delayed neutron precursor is bromine-87, shown below.

$$^{87}_{35}\mathrm{Br} \xrightarrow[55.9\ \mathrm{sec}]{\beta^-} {}^{87}_{36}\mathrm{Kr} \xrightarrow[\text{instantaneous}]{n} {}^{86}_{36}\mathrm{Kr}_{\text{stable}}$$

For most applications, it is convenient to combine the known precursors into groups with appropriately averaged properties. These groups vary somewhat depending on the fissile material in use. Table 3 lists the characteristics for the six precursor groups resulting from thermal fission of uranium-235. The fraction of all neutrons that are produced by each of these precursors is called the delayed neutron fraction for that precursor. The total fraction of all neutrons born as delayed neutrons is called the *delayed neutron fraction* (β). The fraction of delayed neutrons produced varies depending on the predominant fissile nuclide in use. The delayed neutron fractions (β) for the fissile nuclides of most interest are as follows: uranium-233 (0.0026), uranium-235 (0.0065), uranium-238 (0.0148), and plutonium-239 (0.0021).

TABLE 3
Delayed Neutron Precursor Groups
for Thermal Fission in Uranium-235

Group	Half-Life (sec)	Delayed Neutron Fraction	Average Energy (MeV)
1	55.7	0.00021	0.25
2	22.7	0.00142	0.46
3	6.2	0.00127	0.41
4	2.3	0.00257	0.45
5	0.61	0.00075	0.41
6	0.23	0.00027	-
Total	-	0.0065	-

Neutron Generation Time

The neutron generation time is the time required for neutrons from one generation to cause the fissions that produce the next generation of neutrons. The generation time for prompt neutrons (ℓ^* - pronounced "ell-star") is the total time from birth to rebirth. Three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the now thermal neutron exists prior to absorption in fuel, and (c) the time required for a fissionable nucleus to emit a fast neutron after neutron absorption.

Fast neutrons slow to thermal energies or leak out of the reactor in 10^{-4} seconds to 10^{-6} seconds, depending on the moderator. In water moderated reactors, thermal neutrons tend to exist for about 10^{-4} seconds before they are absorbed. Fission and fast neutron production following neutron absorption in a fissionable nucleus occurs in about 10^{-13} seconds. Thus, fast reactors have an ℓ^* of about 10^{-6} seconds, while thermal reactors have an ℓ^* of about 10^{-6} seconds + 10^{-4} seconds, which is about 10^{-4} seconds to 10^{-5} seconds.

On the other hand, the average generation time for the six delayed neutron groups is the total time from the birth of the fast neutron to the emission of the delayed neutron. Again, three time intervals are involved: (a) the time it takes a fast neutron to slow down to thermal energy, (b) the time the thermal neutron exists prior to absorption, and (c) the average time from neutron absorption to neutron emission by the six precursor groups. The average time for decay of precursors from uranium-235 is 12.5 seconds. The other terms in the delayed neutron generation time are insignificant when compared to this value, and the average delayed neutron generation time becomes ~ 12.5 seconds.

A neutron generation time in the range of 10^{-4} seconds to 10^{-5} seconds or faster could result in very rapid power excursions, and control would not be possible without the dependence upon delayed neutrons to slow down the rate of the reaction. The average generation time, and hence the rate that power can rise, is determined largely by the delayed neutron generation time. The following equation shows this mathematically.

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta) \qquad (2\text{-}12)$$

Example:

> Assume a prompt neutron generation time for a particular reactor of 5×10^{-5} seconds and a delayed neutron generation time of 12.5 seconds. If β is 0.0065, calculate the average generation time.

Solution:

$$\begin{aligned}\text{Time}_{\text{average}} &= \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta) \\ &= (5 \times 10^{-5} \text{ seconds}) (0.9935) + (12.5 \text{ seconds}) (0.0065) \\ &= 0.0813 \text{ seconds}\end{aligned}$$

This example demonstrates the effect delayed neutrons have on the neutron generation time and thus reactor control. If a reactor were to be operated in a sustained chain reaction using only prompt neutrons ($\beta = 0$), the generation time from the previous example would be about 5×10^{-5} seconds. However, by operating the reactor such that a 0.0065 fraction of neutrons are delayed, the generation life time is extended to 0.0813 seconds, providing time for adequate operator control. Therefore, although only a small fraction of the total neutron population, delayed neutrons are extremely important to the control and maintenance of a sustained fission chain reaction.

Summary

The important information in this chapter is summarized on the following page.

Prompt and Delayed Neutrons Summary

- Prompt neutrons are released directly from fission within 10^{-13} seconds of the fission event.
- Delayed neutrons are released from the decay of fission products that are called delayed neutron precursors. Delayed neutron precursors are grouped according to half-life. Half-lives vary from fractions of a second to almost a minute.
- The fraction of neutrons born as delayed neutrons is different for different fuel materials. Following are values for some common fuel materials.

Uranium-235	0.0065
Plutonium-239	0.0021

- Delayed neutrons are produced by a classification of fission products known as delayed neutron precursors. When a delayed neutron precursor undergoes a β^- decay, it results in an excited daughter nucleus which immediately ejects a neutron. Therefore, these delayed neutrons appear with a half-life of the delayed neutron precursor.
- The delayed neutron generation time is the total time from the birth of the fast neutron to the emission of the delayed neutron in the next generation. Delayed neutron generation times are dominated by the half-life of the delayed neutron precursor. The average delayed neutron generation time is about 12.5 seconds.
- A prompt neutron generation time is the sum of the amount of time it takes a fast neutron to thermalize, the amount of time the neutron exists as a thermal neutron before it is absorbed, and the amount of time between a fissionable nuclide absorbing a neutron and fission neutrons being released. Prompt neutron generation time is about 5×10^{-5} seconds.
- The average neutron generation time can be calculated from the prompt and delayed neutron generation times and the delayed neutron fraction using Equation (2-12).

$$\text{Time}_{\text{average}} = \text{Time}_{\text{prompt}} (1 - \beta) + \text{Time}_{\text{delayed}} (\beta)$$

- Delayed neutrons are responsible for the ability to control the rate at which power can rise in a reactor. If only prompt neutrons existed, reactor control would not be possible due to the rapid power changes.

NEUTRON FLUX SPECTRUM

The number of neutrons that exist at a given energy level varies. A plot of either the fraction of neutrons or the neutron flux at a given energy versus the energy level is called a neutron energy spectrum. The neutron energy spectrum varies widely for different types of reactors.

EO 4.1 STATE the average energy at which prompt neutrons are produced.

EO 4.2 DESCRIBE the neutron energy spectrum in the following reactors:

a. Fast reactor
b. Thermal reactor

EO 4.3 EXPLAIN the reason for the particular shape of the fast, intermediate, and slow energy regions of the neutron flux spectrum for a thermal reactor.

Prompt Neutron Energies

The neutrons produced by fission are high energy neutrons, and almost all fission neutrons have energies between 0.1 MeV and 10 MeV. The neutron energy distribution, or spectrum, may best be described by plotting the fraction of neutrons per MeV as a function of neutron energy, as shown in Figure 2. From this figure it can be seen that the most probable neutron energy is about 0.7 MeV. Also, from this data it can be shown that the average energy of fission neutrons is about 2 MeV. Figure 2 is the neutron energy spectrum for thermal fission in uranium-235. The values will vary slightly for other nuclides.

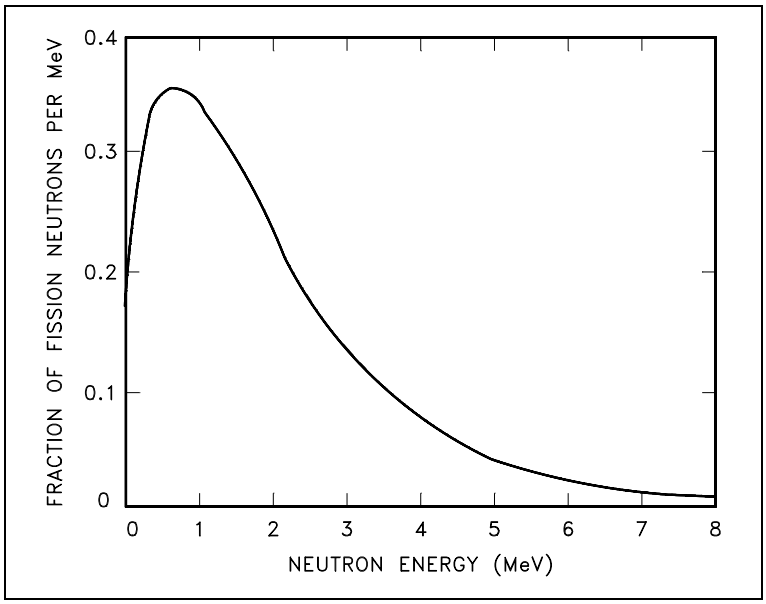


Figure 2 Prompt Fission Neutron Energy Spectrum for Thermal Fission of Uranium-235

Thermal and Fast Breeder Reactor Neutron Spectra

The spectrum of neutron energies produced by fission varies significantly from the energy spectrum, or flux, existing in a reactor at a given time. Figure 3 illustrates the difference in neutron flux spectra between a thermal reactor and a fast breeder reactor. The energy distribution of neutrons from fission is essentially the same for both reactors, so the differences in the curve shapes may be attributed to the neutron moderation or slowing down effects.

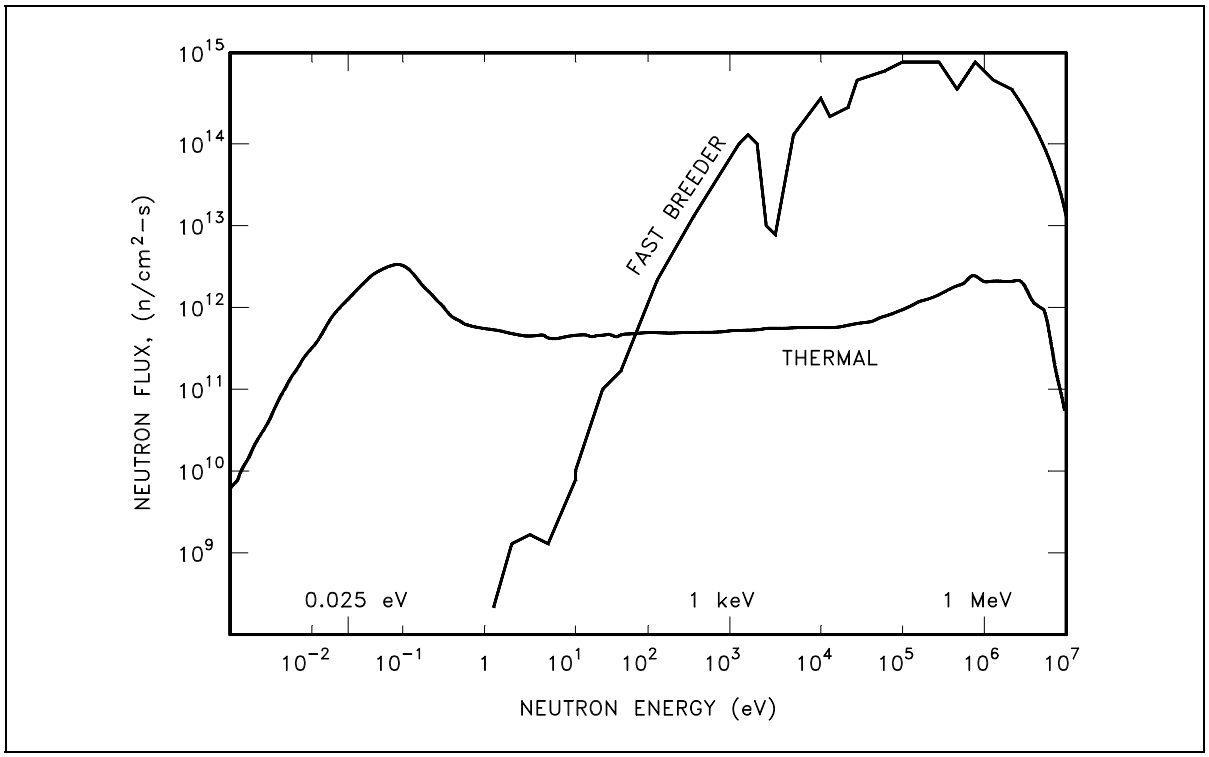


Figure 3 Comparison of Neutron Flux Spectra for Thermal and Fast Breeder Reactor

No attempt is made to thermalize or slow down neutrons in the fast breeder reactor (liquid metal cooled); therefore, an insignificant number of neutrons exist in the thermal range. For the thermal reactor (water moderated), the spectrum of neutrons in the fast region (> 0.1 MeV) has a shape similar to that for the spectrum of neutrons emitted by the fission process.

In the thermal reactor, the flux in the intermediate energy region (1 eV to 0.1 MeV) has approximately a 1/E dependence. That is, if the energy (E) is halved, the flux doubles. This 1/E dependence is caused by the slowing down process, where elastic collisions remove a constant fraction of the neutron energy per collision (on the average), independent of energy; thus, the neutron loses larger amounts of energy per collision at higher energies than at lower energies. The fact that the neutrons lose a constant fraction of energy per collision causes the neutrons to tend to "pile up" at lower energies, that is, a greater number of neutrons exist at the lower energies as a result of this behavior.

In the thermal region the neutrons achieve a thermal equilibrium with the atoms of the moderator material. In any given collision they may gain or lose energy, and over successive collisions will gain as much energy as they lose. These thermal neutrons, even at a specific temperature, do not all have the same energy or velocity; there is a distribution of energies, usually referred to as the Maxwell distribution (e.g., Figure 2). The energies of most thermal neutrons lie close to the most probable energy, but there is a spread of neutrons above and below this value.

Most Probable Neutron Velocities

The *most probable velocity* (v_p) of a thermal neutron is determined by the temperature of the medium and can be determined by Equation (2-13) .

$$v_p = \sqrt{\frac{2\ k\ T}{m}} \tag{2-13}$$

where:

v_p = most probable velocity of neutron (cm/sec)
k = Boltzman's constant (1.38×10^{-16} erg/°K)
T = absolute temperature in degrees Kelvin (°K)
m = mass of neutron (1.66×10^{-24} grams)

Example:

Calculate the most probable velocities for neutrons in thermal equilibrium with their surroundings at the following temperatures. a) 20°C, b) 260°C.

Solution:

a) Calculate the most probable velocity for 20°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2\left(1.38 \times 10^{-16}\ \frac{erg}{°K}\right)(293°K)}{1.66 \times 10^{-24}\ g}}$$

$$= 2.2 \times 10^{5}\ \frac{cm}{sec}\left(\frac{1\ m}{100\ cm}\right)$$

$$= 2200\ \frac{m}{sec}$$

b) Calculate the most probable velocity for 260°C using Equation (2-13).

$$v_p = \sqrt{\frac{2\ k\ T}{m}}$$

$$= \sqrt{\frac{2\left(1.38\ x\ 10^{-16}\ \frac{erg}{°K}\right)(533°K)}{1.66\ x\ 10^{-24}\ g}}$$

$$= 2.977\ x\ 10^{5}\ \frac{cm}{sec}\left(\frac{1\ m}{100\ cm}\right)$$

$$= 2977\ \frac{m}{sec}$$

From these calculations it is evident that the most probable velocity of a thermal neutron increases as temperature increases. The most probable velocity at 20°C is of particular importance since reference data, such as nuclear cross sections, are tabulated for a neutron velocity of 2200 meters per second.

Summary

The important information in this chapter is summarized below.

Neutron Flux Spectrum Summary

- Prompt neutrons are born at energies between 0.1 MeV and 10 MeV. The average prompt neutron energy is about 2 MeV.
- Fast reactors have a neutron energy spectrum that has the same shape as the prompt neutron energy spectrum.
- Thermal reactors have a neutron energy spectrum that has two pronounced peaks, one in the thermal energy region where the neutrons are in thermal equilibrium with the core materials and another in the fast region at energies where neutrons are produced. The flux in the intermediate region (1 eV to 0.1 MeV) has a roughly 1/E dependence.
- The neutron flux spectrum for the fast energy region of a thermal reactor has a shape similar to that of the spectrum of neutrons emitted by the fission process.
- The reason for the 1/E flux dependence at intermediate energy levels in a thermal reactor is due to the neutrons' tendency to lose a constant fraction of energy per collision. Since the neutrons lose a greater amount at the higher energies, the neutrons tend to "pile up" at lower energies where they lose less energy per collision.
- The neutron flux spectrum for the slow region of a thermal reactor contains a peak at the energy where the neutrons are in thermal equilibrium with the atoms of the surrounding material.

Intentionally Left Blank